Supersymmetry and $SU(2) \times U(1)$ breaking with naturally vanishing vacuum energy

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Abstract

We show how the spontaneous breaking of local $N = 1$ supergravity and of the $SU(2) \times U(1)$ gauge symmetry can be simultaneously realized, with naturally vanishing tree-level vacuum energy, in superstring effective supergravities. Both the gravitino mass $m_{3/2}$ and the electroweak scale $M_W$ are classically undetermined, and slide along moduli directions that include the Higgs flat direction $H^0 = [M^2]$. There are important differences with conventional supergravity models: the goldstino has components along the higgsino direction; $SU(2) \times U(1)$ breaking occurs already at the classical level; the scales $m_{3/2}$ and $M_W$, the gauge couplings, the lightest Higgs mass and the cosmological constant are entirely determined by quantum corrections.

1. The gauge hierarchy problem of the Standard Model (SM), linked to the fact that the natural mass scale for an elementary Higgs field is the ultraviolet cutoff, points towards supersymmetric extensions of the SM [1] for describing particle interactions above the electroweak scale. Along this direction, the Minimal Supersymmetric Standard Model (MSSM) should not be considered more than a plausible phenomenological parametrization, possibly useful for organizing experimental searches. A theoretically satisfactory solution of the gauge hierarchy problem requires a model for spontaneous supersymmetry breaking within a more fundamental theory, such as $N = 1$ supergravity, seen in turn as the low-energy limit of a consistent quantum theory at the Planck scale. Besides the gauge hierarchy problem, related to the smallness of the ratio $m_2/M_P \sim 10^{-6}$, in spontaneously broken supergravity there is a second hierarchy problem, associated with the vacuum energy: since the potential is not positive-semidefinite, the natural scale for its VEV after supersymmetry breaking is $\langle V \rangle = O(m_{3/2}^2 M_W^2)$, and one has to explain why it is not so, but instead, after the electroweak and other phase transitions, $\langle V \rangle/M_W^2 \lesssim 10^{-10}$.

Despite intense theoretical efforts over more than a decade, it is not yet clear how the above problems could find a solution. If one tries to make sense of perturbation theory around a flat classical background, avoiding unnatural fine-tunings, a promising starting point are the supergravity models [3,4] characterized by a manifestly positive-semidefinite classical potential, with all minima corresponding to broken supersymmetry and vanishing vacuum energy, and the gravitino mass sliding along some flat direction, parametrized by a gauge-singlet scalar field. At the classical level, the gravitino mass $m_{3/2}$ is undetermined and $SU(2) \times U(1)$ is unbroken, thus the structure of quantum corrections becomes of crucial importance for the viability of such a scenario. At the pure supergravity level, there is no way of controlling $O(m_{3/2}^2 M_W^2)$ contributions to the effective potential, coming from loop corrections in the underlying quantum theory of gravity, which if present would generically forbid both the desired hierarchies. One can at most assume that these contributions are absent, in which case [4] logarithmic quantum corrections can induce a gravitino mass $m_{3/2} \ll M_W$ and break $SU(2) \times U(1)$, with $m_{3/2} = O(m_{3/2})$. Along this line, since 2000s has recently been made in the framework of four-dimensional superstrings, where quantum corrections can be consistently computed and incorporated in the effective supergravity theories: it was possible to identify a restricted class of models [5] where loop contributions to the vacuum energy are computed to be at most $O(m_{3/2}^2)$; however, as we shall describe below, a number of problems were still left unsolved.

To prepare for the following discussion, we would like to recall that, in the above as well as in many other supergravity models, one first discusses supersymmetry breaking in a classical effective supergravity, with a gauge-singlet goldstino belonging to a hidden sector, coupled to the MSSM states via interactions of gravitational strength. Then one takes the flat limit, formally decoupling the hidden sector, and recovers the MSSM with specified forms of its mass parameters. Finally, one computes quantum corrections due to
renormalizable MSSM interactions, to discuss the radiative breaking of $SU(2) \times U(1)$ and the dynamical determination of mass scales.

The most important among the unsolved problems is that of the vacuum energy, which cannot be avoided if one claims to be discussing the low-energy effective theory of a fundamental theory of all interactions, including the gravitational ones. Even if, in the framework of the MSSM, one parametrizes [6] the hidden sector contribution to the cosmological constant with a potential term $\Delta V_{\text{hidd}} = m_{0}^{2} \phi^{2}$, this is not adequate to ensure the vanishing of the vacuum energy generated by the breaking of $SU(2) \times U(1)$: the gravitino mass is a dynamical variable, and the minimization condition with respect to $m_{0}^{2}$, $(\partial V/\partial m_{0}^{2}) = 0$, is not necessarily compatible with the condition of vanishing vacuum energy, $V = 0$. In general, one expects a non-vanishing vacuum energy $\mathcal{O}(m_{0}^{2})$ to be generated, which is unlikely to be cancelled by phase transitions involving much lower mass scales.

Another theoretical, more technical problem of the previous approach is the consistent inclusion of quantum corrections. In principle, these quantum corrections must be computed in the fundamental four-dimensional string theory. Unfortunately, only a very restricted class of models for supersymmetry breaking has been consistently formulated at the string level: for the moment, we are bound to the orbifold models in which supersymmetry is spontaneously broken at the string tree level [7]. Moreover, despite many recent developments [8] in the computation of string loop corrections to the defining functions of the low-energy effective supergravity, complete results, including all the relevant field-dependences, are not yet available. The situation is even worse for the models of supersymmetry breaking involving non-perturbative phenomena: until now, only some models based on gaugino condensation have been formulated only at the level of the effective supergravity theories [9], whilst others [10] have been thoroughly discussed only at the level of global supersymmetry.

Finally, there is a set of more phenomenological cosmological problems [11], associated with gravitationally interacting particles with masses at the electroweak scale or below (the gravitino and the spin 0 and the spin 1/2 components of some singlet moduli fields). Such particles may overclose the universe if they are stable, whereas they may alter the light elements abundance if they decay after primordial nucleosynthesis. For spin 0 fields, there is the additional problem of the energy density stored in the coherent oscillations of the classical fields, which can exceed the closure density if it is not dissipated fast enough.

In view of the above problems, it might be interesting to look for supergravity models in which supersymmetry and $SU(2) \times U(1)$ are both spontaneously broken at the classical level, with naturally vanishing vacuum energy. Once realized at the string level, they could be the starting point for a systematic investigation of perturbative quantum corrections, searching for symmetry properties that might allow for the desired values of $m_{0}^{2}/M_{P}$ and $(V)/M_{P}^{4}$. In particular, one could envisage situations in which the goldstino has signifi-

2The combined breaking of supersymmetry and of a grand-unified gauge symmetry was previously considered in [12].

3We use the standard supergravity mass units where $M_{P} = G_{N}^{-1/2}/\sqrt{8\pi} = 1$.

4The conventionally normalized $U(1)$ coupling is given, as usual, by $g^{2} = (3/5)g_{\mu}^{2}$.
On the other hand, this model exhibits some remarkable properties of the classical effective supergravities [14] corresponding to four-dimensional superstrings [15]. Indeed, it can be identified with a consistent truncation of the string model with spontaneously broken $N = 1$ supersymmetry discussed in [16] (flat directions that break the gauge symmetry were not explicitly investigated there, whereas here we are putting to zero some extra fields that play no role in the symmetry-breaking phenomena under consideration). The singlet fields $S$ and $T$ can be identified with some of the moduli fields: $S$ in the universal dilaton-axion multiplet, parametrizing the Kähler manifold $SU(1,1)/U(1)$; $T$ can be identified with the diagonal combination of the $(1, 1)$ and $(1, 2)$ moduli associated with one complex internal dimension, and $T(\alpha)^{1/2}$ should not be taken too close to $1$ for eqs. (1)–(4) to be a good approximation. As in fermionic constructions [17] and in some orbifold models [18], the fields $(T, H_T^1, H_T^2, \ldots)$, transforming in real representations of $G_0$, parameterize an $SO(2, n)/(SO(2) \times SO(n))$ Kähler manifold: the gauge-invariant parametrization for the Kähler potential of the Higgs fields is taken from [19] and can be obtained from the one of [16], $Y = (x_1 + i x_2)^a - \sum_{a=1}^n (x_1 + i x_2)^a$, by making the field redefinitions $T = x_1 + i x_2, \ H_T^1 = x_1 - i x_2, \ldots$. For the fields $z^a$, transforming in chiral representations of $G_0$, we shall consider only small fluctuations around $z = 0$, so that according to [16] we can consistently assume canonical kinetic terms. As for the superpotential, the cubic couplings are typical of a large class of four-dimensional superstring models, also in the limit of exact supersymmetry, whereas the constant $k$ is the peculiar source of spontaneous supersymmetry breaking in the string constructions of the type considered in [7]. At the level of the classical effective theory, it is not restrictive to choose $k = 1$, since this amounts to a trivial rescaling of the fields $T, H_T^1, H_T^2$.

The scalar potential of the model under consideration can be computed from the general expression

$$ V = V_F + V_D = \frac{1}{2^g \sqrt{g^*}} \left[ C^g (G^{-1})^g G_J - \frac{1}{2} \left( Re f^{-1} \right)_{ab} \left[ C^a (T^a)^g \right] \left[ C^g (T^b)^g \right] \right], $$

where $G \equiv K + \log |w|^2, \ f^g \equiv (S, T, H_T^1, H_T^2, \ldots, z^a)$, and we use standard supergravity conventions on derivatives. After some simple calculation, and keeping only terms up to second order in the fields $z^a$, we find

$$ V_F = \frac{k^2}{2|S + T|^2} \left( |z^a|^2 + \frac{1}{k} \left( h_{00}^{(0)} H_T^0 + h_{00}^{(0)} H_T^0 \right) |z^a|^2 \right) + \frac{1}{2k} \left[ \left( h_{00}^{(0)} (H_T^0 - T) + h_{22}^{(2)} (H_T^2 - T) \right) z^a \bar{z}^a + h.c. \right] + O(z^a), $$

$$ V_D = \frac{k^2 g^g}{2^g} \left( |H_T^0|^2 - |H_T^2|^2 \right)^2 + \frac{|H_T^0|^2 - |H_T^2|^2}{2 Y} \left( f^g (f^{g*})^g \right) |z^a|^2 + O(z^a). \quad \text{(7)} $$

Notice that, for $z^a = 0$, $V_F$ vanishes identically, and the same holds for the positive-semidefinite term $V_D$ along directions $|H_T^0|^2 = |H_T^2|^2$. In other words, $(V) = 0$ for $|z^a| = 0$ and for arbitrary values of $(S), (T)$, and $(|H_T^0|^2) = (|H_T^2|^2)$. Thus the model has a classically degenerate set of vacua where the cosmological constant vanishes and, at the same time, both the $SU(2) \times U(1)$ gauge symmetry and local $N = 1$ supersymmetry are spontaneously broken. With the definitions $s \equiv (S + T), \ t \equiv (T + T)$ and $x \equiv (H_T^1 + H_T^2)$, the gravitino mass reads

$$ m_{3/2}^2 = \left( e^{c_{3/2}} \right)^2 = \frac{k^2}{s (\beta - |x|^2)}. \quad \text{(8)} $$

To obtain the physical mass spectrum in the remaining sectors of the model, one must take into account the presence of non-canonical kinetic terms. In particular, the dilaton-axion $S$ and the gauge superfields can be normalized via a simple rescaling, whereas a non-trivial mixing occurs in the $(T, H_T^1, H_T^2)$ sector, due to the more complicated Kähler metric.

In the gauge boson sector, putting $\rho \equiv (|H_T^0|^2) = (|H_T^2|^2)$, we get

$$ m_{\rho}^2 = \left( g^g + \rho^g \right) \frac{\beta}{1 - |x|^2}, \quad m_{\rho}^2 = \frac{2}{\rho} \left( g^g + \rho^g \right) \frac{\beta}{1 - |x|^2}. \quad \text{(9)} $$

Observing that, depending on the relative phase of $(H_T^0)$ and $(H_T^2)$, $0 \leq |x|^2 \leq 4 \beta^2$, we can see that it should be $\rho^g = 0.25 \times 10^{-32}$ to reproduce the experimentally measured values of $m_{\omega}$ and $m_{\xi}$, irrespective of the individual values of $|\rho|^2, \rho^g \approx \rho^g$ and $\beta$. It is interesting to observe that, for $|H_T^0|^2 = -|H_T^2|^2$, one can have $\rho^g = 0$ with $x = 0$: this allows for a situation in which the gravitino mass (and the volume of moduli space) do not depend on the VEV $\rho$ which breaks $SU(2) \times U(1)$.

In the scalar sector, both spin-0 components of $S$ are massless. Among the five physical real degrees of freedom of the $(T, H_T^1, H_T^2)$ sector, which remain after removing the neutral Goldstone boson associated with the $Z$, four are massless and one has mass $m_T$. Observing that the charged Higgs fields must appear in the $V$ function of eq. (3) via the combination $-(H_T^1 - H_T^2)(H_T^1 - H_T^2)$, it is easy to verify that the charged Higgs sector contains, besides the physical charged Goldstone boson, a physical state with mass $m_{\omega}$.

To discuss the masses of the bosonic and fermionic components of the 'matter' superfields $z^a$, we neglect intergenerational mixing, rewriting the superpotential couplings involving the neutral Higgs fields in the simplified form

$$ \frac{1}{2} h_{00}^{(0)} z^a \bar{z}^a H_T^0 + \frac{1}{2} h_{22}^{(2)} z^a \bar{z}^a H_T^2 = \sum_T h_T^{(0)} f^g H_T^0 + \sum_T h_T^{(2)} f^g H_T^2. \quad \text{(10)} $$

Then the scalar fields have diagonal masses

$$ m_{\rho}^{2} = m_{\rho}^{2} + m_{\rho}^{2}, \quad m_{\rho}^{2} = m_{\rho}^{2} + m_{\rho}^{2}, \quad m_{\rho}^{2} = m_{\rho}^{2} + m_{\rho}^{2}, \quad m_{\rho}^{2} = m_{\rho}^{2} + m_{\rho}^{2}, \quad \text{where} \quad m_{\rho}^{2} = \frac{h_T^{22} \rho^g}{\sqrt{\beta - |x|^2}}, \quad m_{\rho}^{2} = \frac{h_T^{22} \rho^g}{\sqrt{\beta - |x|^2}}. \quad \text{(11)} $$

$$ \text{(12)} $$
are the masses of the corresponding fermions. In general, there are also off-diagonal scalar mass terms of the form
\[ m^2_{4}\phi = \frac{h_4((H_1^0) - (H_2^0))}{\sqrt{6(\phi^2 - |x|^2)}} m_{3/2}, \quad m^2_{j}\phi^* = \frac{h_4((H_1^0) - (H_2^0))}{\sqrt{6(\phi^2 - |x|^2)}} m_{3/2}. \tag{13} \]

The squared mass matrix in the chargino sector reads
\[ \begin{pmatrix} m^2_{3/2} + m^2_\chi & m_{3/2}m_W \langle H_1^0 - H_2^0 \rangle_f \\ m_{3/2}m_W \langle H_1^0 - H_2^0 \rangle_f & m^2_\chi + m^2_\phi \end{pmatrix}, \tag{14} \]
with eigenvalues \( m^2_{3/2} + m^2_\chi \pm m_{3/2}m_W \langle H_1^0 - H_2^0 \rangle_f / \rho \). For \( \langle H_1^0 \rangle = \langle H_2^0 \rangle \), one gets two degenerate states of mass \( \sqrt{m^2_{3/2} + m^2_\chi} \), whereas the maximum mass splitting is obtained for \( \langle H_1^0 \rangle = -\langle H_2^0 \rangle \), in which case the two masses are \( |m_{3/2} \pm m_W| \).

In the neutralino sector (which includes here one more physical state than in the MSSM), a linear combination of the fermionic S and \( (T, H_1^0, H_2^0) \) fields can be identified with the goldstino. The three remaining physical states mix between them and with the two neutral electroweak gauginos (the gluino mass is equal to the gravitino mass). The corresponding masses satisfy the sum rule \( \sum_{i=1}^{3} m_i^2 = m_{3/2}^2 + 2m_\chi^2 \).

To understand the structure of the present model better, it is convenient to take the limit \( \rho / t \rightarrow 0 \), which leads to a conventional supergravity model with hidden sector and, when interactions of gravitational strength are neglected, to a special version of the MSSM. In such a limit, the goldstino becomes a linear combination of the S and T fermions only, with mixing angle, in the notation of \( [20] \), \( \sin^2 \theta = 1/3 \). The orthogonal combination has mass \( m_{3/2} \). The MSSM mass parameters take the special values
\[ m^2_{3/2} = m_{3/2}^3, \quad m^2_{\text{matter}} = m^2_{3/2}, \quad m^2_{\text{Higgs}} = -m^2_{3/2}, \quad \mu^2 = m^2_{3/2}, \quad A^2 = m^2_{3/2}, \quad B = 0, \tag{15} \]
as can be easily checked by looking at the limiting form of the supergravity mass matrices, remembering that in the chosen limit the canonically normalized Higgs fields are given by \( H_1^0 / t \), and the MSSM Yukawa couplings by \( h_{1,4}/\sqrt{6} \). Notice in particular that the MSSM mass terms exhibit remarkable universality properties, much more stringent than usually assumed in the general MSSM framework, with one important exception: since the kinetic terms for the Higgs and matter fields have different scaling properties with respect to the t modulus, the corresponding soft scalar masses have different values. In particular, the standard mass parameters of the classical MSSM Higgs potential are given by \( m_1^2 = m_2^2 = m_3^2 = 0 \), which allows for \( SU(2) \times U(1) \) breaking along the flat direction \( \langle H_1^0 \rangle = \langle H_2^0 \rangle \).

3. In conclusion, we have constructed a semi-realistic supergravity model in which, already at the classical level, both \( N = 1 \) supersymmetry and the \( SU(2) \times U(1) \) gauge symmetry are spontaneously broken with naturally vanishing vacuum energy, thanks to the remarkable geometrical properties of superstring effective supergravities.

We would like to stress again some important differences with the supergravity models considered so far in the literature. Usually, \( SU(2) \times U(1) \) is unbroken at the classical level, and the goldstino is neutral under \( SU(2) \times U(1) \), to break \( SU(2) \times U(1) \) one appeals to radiative corrections. In the present model, \( SU(2) \times U(1) \) breaking occurs already at the classical level, and the goldstino has non-vanishing components along the neutral higgsino directions.

Due to the role played by the internal singlet modulus \( T \), which takes part in the superhiggs mechanism and forces the ratio \( \rho / t \) to be of order \( m_2 / M_P \), in order to reproduce the experimental value of the electroweak scale, the interactions of the gravitino via its goldstino (higgsino) components are suppressed down to gravitational strength. However, it is conceivable that one could construct similar models where \( S \) and \( T \) do not take part in the superhiggs mechanism, and all the light mass eigenstates have interactions of electroweak strength. In such a framework, some cosmological problems of gravitationally-coupled states with electroweak-scale masses would be avoided (even if others could arise). We have not yet been able to build a model with these properties. To do so, one should probably move to models where the spontaneously broken gauge group \( SU(2) \times U(1) \) can be embedded into the ‘compensator’ subgroup of the isometry group of the manifold containing the Higgs fields. The manifold \( SO(2, n)/[SO(2) \times SO(n)] \) is not a viable candidate, whereas manifolds such as \( SU(3)/[SU(3) \times SU(n) \times U(1)] \) or \( SU(2)/[SU(2) \times SU(n) \times U(1)] \), associated to some untwisted sectors of \( Z_5 \) and \( Z_6 \) orbifold models, might be viable\(^6\).

The tree-level potential of our model exhibits several flat directions, associated to classically massless scalar fields, some of which control, via their VEVs, the gauge- and supersymmetry-breaking scales and the dimensionless couplings. Generically, we expect these flat directions to be removed by the consistent inclusion of quantum corrections, which should fix the VEVs along the flat directions and the corresponding scalar masses. However, it is difficult to make definite statements about the actual values of these VEVs and masses, since expectations based on simple dimensional arguments may turn out to be incorrect. For example, in the models of ref. [5] the singlet scalar partners of the goldstino have masses of order \( m_{3/2} / M_P \), and not \( m_{3/2} \), because of the absence of loop contributions to the vacuum energy of order \( m_{3/2}^2 M_P^2 \). Similarly, if loop corrections fix \( m_W \) to its experimental value, and CP is not spontaneously broken, loop contributions to the lightest CP-even Higgs boson mass associated with top and stop loops \([21]\) are of order \( m_{3/2}^2 m_W \), which already brings the latter into the phenomenologically acceptable region. It might even be that some direction remains flat, as assumed for example in the recent

\(^6\)We thank C. Kounnas for a clarifying discussion on this point.
work by Polyakov and Damour [22], who discussed possible phenomenological implications of a massless dilaton. Consistent inclusion of perturbative quantum corrections would require the knowledge not only of the singlet-moduli dependence, but also of the Higgs-field dependence of the string loop corrections to the low-energy effective supergravity. This is not fully available yet, but it is not inconceivable that it will be soon, at least in some simple classes of four-dimensional string constructions. In this case, many more interesting questions could be addressed, including a quantitative study of the dynamical determination of the gravitino and electroweak scales.

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