Controlling Rescattering Effects in Constraints on the
CKM Angle $\gamma$ arising from $B \to \pi K$ Decays

Robert Fleischer
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

It has recently been pointed out that the observables of the decay $B^+ \to K^+\overline{K}^0$ and its charge conjugate allow us to take into account rescattering effects in constraints on the CKM angle $\gamma$ arising from $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\pm K^\pm$ modes, and that they play an important role to obtain insights into final-state interactions. In this paper, the formalism needed to accomplish this task is discussed in detail. Furthermore, using a transparent model to describe the rescattering processes, as well as electroweak penguins, we calculate the quantities parametrizing the $B^+ \to \pi^+ K^0$ and $B^0_d \to \pi^- K^+$ decay amplitudes for specific examples, and illustrate the constraints on $\gamma$ arising from the corresponding observables. Although this model is very crude, it shows nicely the power of $B^\pm \to K^\pm K$ both to include the rescattering effects in the bounds on $\gamma$ and to obtain insights into final-state interactions. Moreover this model exhibits the interesting feature that the combined branching ratio $BR(B^\pm \to K^\pm K)$ may be considerably enhanced through rescattering processes, as was recently pointed out within a general framework.
1 Introduction

The issue of rescattering effects in $B \to \pi K$ decays, originating from processes such as $B^+ \to \{\pi^0 K^+\} \to \pi^+ K^0$, led to considerable interest in the recent literature [1]-[5] (for earlier references, see [6]). An important implication of these final-state interaction effects may be direct CP violation, in the mode $B^+ \to \pi^+ K^0$, as large as $\mathcal{O}(10\%)$, whereas estimates performed at the perturbative quark level, following the approach proposed by Bander, Silverman and Soni [7], typically give CP asymmetries of at most a few percent [8]. Several papers dealing with these rescattering effects tried to point out that they would also invalidate bounds on the angle $\gamma$ of the usual non-squashed unitarity triangle [9] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [10] that were derived in [11]; they would arise if the ratio

$$R \equiv \frac{\text{BR}(B_d \to \pi^+ K^\pm)}{\text{BR}(B^\pm \to \pi^\pm K)}$$

of the combined branching ratios

$$\text{BR}(B_d \to \pi^\mp K^\pm) \equiv \frac{1}{2} \left[ \text{BR}(B_d^0 \to \pi^- K^+) + \text{BR}(\bar{B}_d^0 \to \pi^+ K^-) \right]$$

$$\text{BR}(B^\pm \to \pi^\pm K) \equiv \frac{1}{2} \left[ \text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0) \right]$$

is found to be smaller than 1. These quantities have recently been measured for the first time by the CLEO collaboration, and the present experimental results are as follows [12]:

$$\text{BR}(B_d \to \pi^\mp K^\pm) = \left(1.5^{+0.5}_{-0.4} \pm 0.1 \pm 0.1\right) \times 10^{-5}$$

$$\text{BR}(B^\pm \to \pi^\pm K) = \left(2.3^{+1.1}_{-1.0} \pm 0.3 \pm 0.2\right) \times 10^{-5},$$

yielding $R = 0.65 \pm 0.40$. Consequently, it may well be that future measurements will stabilize at a value of $R$ that is significantly smaller than 1, thereby leading to interesting constraints on the CKM angle $\gamma$ [11] (for a detailed study, see [13]). At first sight, an important limitation of the theoretical accuracy of these bounds is in fact due to the rescattering effects mentioned above [2]-[5]. A closer look shows, however, that these effects do not spoil the bounds on $\gamma$, and can be included completely, in a rather straightforward way, through the decay $B^+ \to K^+ K^0$ and its charge conjugate, providing in addition valuable insights into final-state interactions [14]. The combined branching ratio $\text{BR}(B^\pm \to K^\pm K)$ for this decay, which is defined in analogy to (2) and (3), is very sensitive to rescattering processes and may be enhanced considerably through them.

In this paper, we illustrate these interesting features in more detail by following closely [14] and using the parametrization of the $B^+ \to \pi^+ K^0$ and $B_d^0 \to \pi^- K^+$ decay amplitudes in terms of the “physical” quantities given there. The outline is as follows: in Section 2, we collect the expressions for the $B \to \pi K$ decay amplitudes, introduce the relevant observables, and discuss briefly the constraints on the CKM angle $\gamma$ implied by them.
In Section 3, we focus on the approach to take into account rescattering effects in these bounds with the help of $B^\pm \to K^\pm K$. This strategy, as well as the realization of the constraints on $\gamma$, is illustrated in a quantitative way in Section 4, where we will use a simple model to describe the rescattering effects that has been proposed in [2, 3] and is based on the assumption of elastic final-state interactions. In Section 5, a few concluding remarks are given.

2 Decay Amplitudes, Observables and Constraints on the CKM Angle $\gamma$

The general $B^+ \to \pi^+ K^0$ and $B^0_d \to \pi^- K^+$ decay amplitudes arising within the framework of the Standard Model can be expressed as follows [14]:

$$ A(B^+ \to \pi^+ K^0) = P $$

$$ A(B^0_d \to \pi^- K^+) = -[P + T + P_{ew}] . $$

In order to derive these amplitude relations, the $SU(2)$ isospin symmetry of strong interactions has been used. The amplitude $P$, which is usually referred to as a “penguin” amplitude, takes the form

$$ P = -\left(1 - \frac{\lambda^2}{2}\right)\lambda^2 A \left[1 + \rho e^{i\theta} e^{i\gamma}\right] P_{tc} , $$

where

$$ \rho e^{i\theta} = \frac{\lambda^2 R_b}{1 - \lambda^2/2} \left[1 - \left(\frac{P_{uc}}{P_{tc}} + A\right)\right] . $$

Here the quantities $P_{tc} \equiv |P_{tc}| e^{i\delta_{tc}}$ and $P_{uc} \equiv |P_{uc}| e^{i\delta_{uc}}$ describe contributions originating from penguin topologies with internal top and charm, and up and charm quarks, respectively, $A$ is due to annihilation processes, and

$$ \lambda \equiv |V_{us}| = 0.22 , \quad A \equiv \frac{1}{\lambda^2} |V_{cb}| = 0.81 \pm 0.06 , \quad R_b \equiv \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right| = 0.36 \pm 0.08 $$

are the relevant CKM factors, expressed in terms of the Wolfenstein parameters [15]. The amplitudes $T$ and $P_{ew}$ – the latter is due to electroweak penguins – can be parametrized in a simple way as

$$ T \equiv |T| e^{i\delta_T} e^{i\gamma_T} , \quad P_{ew} \equiv -|P_{ew}| e^{i\delta_{ew}} , $$

where $\delta_T$ and $\delta_{ew}$ are CP-conserving strong phases such as $\delta_{tc}$, $\delta_{uc}$ and $\theta$. The expressions for the charge-conjugate decays $B^- \to \pi^- K^0$ and $B^0_d \to \pi^- K^-$ can be obtained straightforwardly from (6) and (7) by performing the substitution $\gamma \to -\gamma$ in (8) and (11), i.e. $|T|$ and $|P_{ew}|$, in contrast to $|P|$, exhibit no CP violation. In the literature, $T$ is usually referred to as a “tree” amplitude. This terminology is, however, misleading in
In order to obtain information on the CKM angle $\gamma$, in addition to the ratio $R$ of the combined $B \rightarrow \pi K$ branching ratios introduced in (1), the “pseudo-asymmetry”

$$A_0 \equiv \frac{\text{BR}(B^0_d \rightarrow \pi^- K^+) - \text{BR}(\bar{B}^0_d \rightarrow \pi^+ K^-)}{\text{BR}(B^+ \rightarrow \pi^0 K^0) + \text{BR}(B^- \rightarrow \pi^- \bar{K}^0)} = A_{\text{CP}}(B_d \rightarrow \pi^\pm K^\mp)R,$$  

(12)

as well as the quantities

$$r \equiv \frac{|T|}{\sqrt{\langle |P|^2 \rangle}}, \quad \epsilon \equiv \frac{|P_{\text{ew}}|}{\sqrt{\langle |P|^2 \rangle}},$$

(13)

where $\langle |P|^2 \rangle$ is defined by

$$\langle |P|^2 \rangle \equiv \frac{1}{2} (|P|^2 + |\bar{P}|^2)$$

(14)

and

$$\delta \equiv \delta_T - \delta_c, \quad \Delta \equiv \delta_{\text{ew}} - \delta_c$$

(15)

are differences of the relevant strong phases, turn out to be very useful [14]. The general expressions for $R$ and $A_0$ in terms of the parameters specified in (13) and (15) are quite complicated and are given explicitly in [14], where further details can be found. Let us here just briefly discuss the basic ideas that are at the basis of the constraints on the CKM angle $\gamma$ arising from these observables.

The pseudo-asymmetry $A_0$ allows us to eliminate the strong phase $\delta$ in the expression for $R$. Consequently, if both $R$ and $A_0$ have been measured, contours in the $\gamma-r$ plane can be fixed. If the parameter $r$, i.e. $|T|$, could also be fixed, we were in a position to extract the value of $\gamma$ from these contours up to a four-fold ambiguity [16, 17]. Unfortunately, since $T$ is not just a “tree” amplitude, $r$ may in general receive sizeable non-factorizable contributions. Therefore, expectations relying on “factorization” that a future theoretical accuracy of $r$ as small as $\mathcal{O}(10\%)$ may be achievable (see, for instance, [17, 18]) appear too optimistic.

It is, however, in principle possible to constraint the CKM angle $\gamma$ in a way that does not depend on $r$, introducing the major theoretical uncertainty into the extraction of $\gamma$ sketched in the previous paragraph. Provided $R$ turns out to be smaller than 1, an interval around $\gamma = 90^\circ$ can be ruled out [11], which is of particular phenomenological importance [13]. As soon as a non-vanishing value of $A_0$ has been measured, also regions for $\gamma$ around $0^\circ$ and $180^\circ$ can be excluded. These constraints on $\gamma$ are related to the fact that $R$ (considered as a function of $r$; $\delta$ has been eliminated through $A_0$) takes a minimal value, which is given by [14]

$$R_{\text{min}} = \kappa \sin^2 \gamma + \frac{1}{\kappa} \left( \frac{A_0}{2 \sin \gamma} \right)^2,$$  

(16)

where

$$\kappa = \frac{1}{w^2} \left[ 1 + 2 (\epsilon w) \cos \Delta + (\epsilon w)^2 \right] \quad \text{with} \quad w = \sqrt{1 + 2 \rho \cos \theta \cos \gamma + \rho^2}.$$  

(17)
In particular, the lower bound

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conjugate... In order to go beyond these constraints, the decay has been measured, we are in a position to obtain upper and lower bounds on processes and by contributions from electroweak penguins, which are included in $\kappa$ through the parameters $\rho$ and $\epsilon$, respectively. Neglecting these effects, we simply have $\kappa = 1$, which corresponds to the case discussed in the original paper [11] on the bounds on $\gamma$ arising from $B \to \pi K$ decays. Let us focus in the following section on the rescattering processes, which have led to considerable interest in the recent literature [2]–[5]. A detailed discussion of electroweak penguin effects can be found in [14] (see also [3, 17]).

### 3 Controlling the Rescattering Effects

The parameter $\rho$ describing the “strength” of the rescattering processes is highly CKM-suppressed by $\lambda^2 R_0 \approx 0.02$, as can be seen in (9). Model calculations performed at the perturbative quark level give $\rho = \mathcal{O}(1\%)$ and do not indicate a significant compensation of this very large CKM suppression. However, in a recent attempt [4] to evaluate rescattering processes of the kind $B^+ \to \{\pi^0 K^+, \pi^0 K^{++}, \rho^0 K^{*+}, \ldots\} \to \pi^+ K^0$, it is found that $|\mathcal{P}_{uc}|/|\mathcal{P}_{tc}| = \mathcal{O}(5)$, implying that $\rho$ may be as large as $\mathcal{O}(10\%)$. A similar feature arises also in a simple model to describe final-state interactions, which has been proposed in [2, 3]. We will use this model for illustrative purposes in Section 4, where it is discussed in more detail.

Although it has been claimed by several authors in recent literature that such rescattering processes would invalidate the constraints on the CKM angle $\gamma$ implied by the $B_d \to \pi^\pm K^\mp$, $B^\pm \to \pi^\pm K$ observables, this is actually not the case [14]. These effects can be included in the bounds on $\gamma$ by using additional experimental data. The purpose of this section is to give a detailed discussion of this important feature, which will be illustrated in a quantitative way in the following section.

A first step towards the control of rescattering processes is provided by the CP-violating asymmetry

$$A_+ \equiv \frac{\text{BR}(B^+ \to \pi^+ K^0) - \text{BR}(B^- \to \pi^- K^0)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)} = -\frac{2 \rho \sin \theta \sin \gamma}{1 + 2 \rho \cos \theta \cos \gamma + \rho^2}. \quad (18)$$

While simple quark-level estimates give at most a few percent for this CP asymmetry [8], rescattering processes may lead to values as large as $\mathcal{O}(10\%)$ [2]–[5]. As soon as $A_+$ has been measured, we are in a position to obtain upper and lower bounds on $\rho$, which are given by

$$\rho_{\text{max}} = \frac{\sqrt{A_+^2 + (1 - A_+^2) \sin^2 \gamma} \pm \sqrt{(1 - A_+^2) \sin^2 \gamma}}{|A_+|}. \quad (19)$$

In particular the lower bound $\rho_{\text{min}}$ is of special interest. A detailed study can be found in [14]. In order to go beyond these constraints, the decay $B^+ \to K^+ K^0$ and its charge conjugate – the SU(3) counterparts of $B^\pm \to \pi^\pm K$ – play a key role. The corresponding decay amplitude takes the form

$$A(B^+ \to K^+ K^0) = \lambda^2 A \left[1 - \left(\frac{1 - \lambda^2}{\lambda^2}\right) \rho^{(d)} e^{i\delta_d} e^{i\gamma}\right] \mathcal{P}_{tc}^{(d)}, \quad (20)$$
where
\[ \rho^{(d)} e^{i\theta_d} = \frac{\lambda^2 R_b}{1 - \lambda^2/2} \left[ 1 - \left( \frac{\mathcal{P}_{uc}^{(d)} + A^{(d)}}{\mathcal{P}_{lc}^{(d)}} \right) \right] \] (21)
corresponds to (9), and direct CP violation is described by
\[ A_{+}^{(d)} = \frac{\text{BR}(B^+ \rightarrow K^+\bar{K}^0) - \text{BR}(B^- \rightarrow K^-K^0)}{\text{BR}(B^+ \rightarrow K^+\bar{K}^0) + \text{BR}(B^- \rightarrow K^-K^0)} = \frac{2 \lambda^2 (1 - \lambda^2) \rho^{(d)} \sin \theta_d \sin \gamma}{\lambda^4 - 2 \lambda^2 (1 - \lambda^2) \rho^{(d)} \cos \theta_d \cos \gamma + (1 - \lambda^2)^2 \rho^{(d)}^2} . \] (22)
Moreover, the following ratio of combined branching ratios turns out to be very useful [1]:
\[ H \equiv R_{SU(3)}^2 \left( \frac{1 - \lambda^2}{\lambda^2} \right) \frac{\text{BR}(B^\pm \rightarrow K^\pm K)}{\text{BR}(B^\pm \rightarrow \pi^\pm K)} = \frac{\lambda^4 - 2 \lambda^2 (1 - \lambda^2) \rho^{(d)} \cos \theta_d \cos \gamma + (1 - \lambda^2)^2 \rho^{(d)}^2}{\lambda^4 (1 + 2 \rho \cos \theta \cos \gamma + \rho^2)} . \] (23)
Here \( \text{BR}(B^\pm \rightarrow K^\pm K) \) is defined in analogy to (3), tiny phase-space effects have been neglected (for a more detailed discussion, see [11]), and
\[ R_{SU(3)} = \frac{M_B^2 - M_{\pi}^2}{M_B^2 - M_K^2} \frac{F_{Br}(M_K^2;0^+)}{F_{Br}(M_K^2;0^+)} \] (24)
describes factorizable \( SU(3) \) breaking. Using the model of Bauer, Stech and Wirbel [19] to estimate the relevant form factors, we have \( R_{SU(3)} = O(0.7) \). At present, there is unfortunately no reliable approach available to deal with non-factorizable \( SU(3) \) breaking.

If we look at (18), (22) and (23), we observe that \( A_+, A_{+}^{(d)} \) and \( H \) depend on the four "unknowns" \( \rho, \theta, \rho^{(d)}, \theta_d \), and of course also on the CKM angle \( \gamma \). A possible strategy is to use
\[ \rho = \zeta_\rho \rho^{(d)} , \] (25)
where \( \zeta_\rho \) parametrizes \( SU(3) \)-breaking corrections, in order to express \( \rho^{(d)} \) in (22) and (23) through \( \rho \). As a first "guess", we may use \( \zeta_\rho = 1 \). The strong phase \( \theta_d \) can be eliminated in \( H \) with the help of the CP asymmetry \( A_+^{(d)} \) arising in \( B^\pm \rightarrow K^\pm K \), while \( \theta \) can be eliminated through the CP asymmetry \( A_+ \) arising in \( B^\pm \rightarrow \pi^\pm K \). Following these lines, we arrive at an expression for \( H \), which depends only on \( \rho, \gamma \), and on the \( SU(3) \)-breaking parameter \( \zeta_\rho \). Consequently, specifying \( R_{SU(3)} \) and \( \zeta_\rho \), for instance through \( R_{SU(3)} = 0.7 \) and \( \zeta_\rho = 1 \), we are in a position to determine \( \rho \) and \( \theta \) as functions of \( \gamma \). In order to include the rescattering effects in the contours in the \( \gamma-r \) plane and the bounds on \( \gamma \) arising from (16), \( \rho \) and \( \theta \) determined this way are sufficient [14]. Keeping the \( SU(3) \)-breaking parameters \( R_{SU(3)} \) and \( \zeta_\rho \) explicitly in the corresponding formulae, it is possible to study the sensitivity to their chosen values, and to take into account \( SU(3) \) breaking once we have a better understanding of this phenomenon.
To simplify the following discussion, let us assume
\[ \rho = \rho^{(d)} \quad \text{and} \quad \theta = \theta_d. \] (26)
As has already been pointed out in [1, 14], this SU(3) input implies a nice relation
between \( A_+ \), \( A_+^{(d)} \) and the combined \( B^\pm \to \pi^\pm K \) and \( B^\pm \to K^\pm K \) branching ratios,
which is given by
\[ \frac{A_+}{A_+^{(d)}} = - R_{SU(3)}^{2} \frac{\text{BR}(B^\pm \to K^\pm K)}{\text{BR}(B^\pm \to \pi^\pm K)} = - \left( \frac{\lambda^2}{1 - \lambda^2} \right) H, \] (27)
and allows the determination of \( H \) and of the \( SU(3) \)-breaking parameter \( R_{SU(3)} \) directly
from the measured \( B^\pm \to \pi^\pm K \) and \( B^\pm \to K^\pm K \) observables. Using (23) and (26), it is
an easy exercise to derive the expression
\[ 2 \rho \cos \theta \cos \gamma = a + b \rho^2 \] (28)
with
\[ a = \lambda^2 \left[ \frac{1 - H}{1 + \lambda^2 (H - 1)} \right], \quad b = \frac{1}{\lambda^2} \left[ \frac{(1 - \lambda^2)^2 - \lambda^4 H}{1 + \lambda^2 (H - 1)} \right], \] (29)
which leads to
\[ w = \frac{1}{\lambda} \sqrt{\frac{\rho^2 + \lambda^2 (1 - \rho^2)}{1 + \lambda^2 (H - 1)}}. \] (30)
Combining (28) with the CP-violating asymmetry (18), we obtain a quadratic equation
for \( \rho^2 \), which has the solution
\[ \rho^2 = \frac{w \pm \sqrt{w^2 - uv}}{v}, \] (31)
where
\[ u = \left[ a \sin^2 \gamma + (a + 1) A^2_+ \cos^2 \gamma \right]^2 + \left( A_+ \sin \gamma \cos \gamma \right)^2 \] (32)
\[ v = \left[ b \sin^2 \gamma + (b + 1) A^2_+ \cos^2 \gamma \right]^2 + \left( A_+ \sin \gamma \cos \gamma \right)^2 \] (33)
\[ w = \left[ A^2_+ + 2 \left( 1 - A^2_+ \right) \sin^2 \gamma \right] \sin^2 \gamma \cos^2 \gamma - \left[ a \sin^2 \gamma + (a + 1) A^2_+ \cos^2 \gamma \right] \left[ b \sin^2 \gamma + (b + 1) A^2_+ \cos^2 \gamma \right]. \] (34)

The present upper limit on the combined \( B^\pm \to K^\pm K \) branching ratio obtained by
the CLEO collaboration is given by \( 2.1 \times 10^{-5} \) [12]. At first sight, experimental studies of
this mode appear to be difficult, since the “short-distance” expectation for its combined
branching ratio is \( O(10^{-6}) \) (see, for instance, [8]). However, as was pointed out in [14],
rescattering effects may enhance this observable by a factor as large as \( O(10) \), and could
thereby make \( B^\pm \to K^\pm K \) measurable at future \( B \) factories. In the following section,
we illustrate this feature, as well as the constraints on \( \gamma \) and the strategy to control the
rescattering processes affecting them, in a quantitative way, by using a simple model.
4 An Illustration within a Simple Model

In Ref. [2], a simple model was introduced to deal with final-state interactions in $B \to \pi K$ decays, which has been refined in [3], where also electroweak penguin effects are considered. The basic idea of this model is very simple: the generalized factorization prescription [20] is used to calculate the “short-distance” contributions to the $B \to \pi K$ decay amplitudes, whereas the “long-distance” effects are taken into account by simply introducing elastic rescattering phases $\phi_{1/2}$ and $\phi_{3/2}$ for the two isospin channels of the final-state mesons. Following these lines, we obtain

$$A(B^+ \to \pi^+ K^0) = \left[ e^{i\phi_p} + \frac{1}{3} z \left( 1 + \left( 1 + \frac{1}{y_{ew}} \right) \left( e^{i\Delta \phi} - 1 \right) \right) \right] - \frac{1}{3} x (1 + y) \left( e^{i\Delta \phi} - 1 \right) e^\gamma |M_P| e^{i\phi_{1/2}}$$

$$A(B_d^0 \to \pi^- K^+) = \left[ e^{i\phi_p} - \frac{1}{3} z \left( 2 + \left( 1 + \frac{1}{y_{ew}} \right) \left( e^{i\Delta \phi} - 1 \right) \right) \right] + x \left( 1 + \frac{1}{3} (1 + y) \left( e^{i\Delta \phi} - 1 \right) \right) e^\gamma |M_P| e^{i\phi_{1/2}},$$

where $\Delta \phi = \phi_{3/2} - \phi_{1/2}$ is the difference of the elastic rescattering phases, and $\phi_P$ has been introduced to describe the strong phase of penguin topologies with internal charm quarks, which receive important contributions from rescattering processes of the kind $B^+ \to \{D^- D_s^+, \bar{D}^0 D_s^{*+}, \bar{D}^{*0} D_1^{*+}, \ldots \} \to \pi^+ K^0$. Neglecting such effects, $\phi_P$ takes the trivial value $180^\circ$, which is related to the minus sign appearing in (8). In this simple model, final-state interactions can be “switched on” by choosing a non-vanishing value for $\Delta \phi$. If we denote the colour-allowed and colour-suppressed “tree” amplitudes obtained within the framework of generalized factorization [3, 20] by $M_T$ and $M_C$, respectively, and the corresponding $b \to s$ QCD penguin amplitude by $M_P$, the parameters $x$ and $y$ are given by

$$x = \frac{|M_T|}{|M_P|} \approx 0.2, \quad y = \frac{|M_C|}{|M_T|} \approx \frac{a_2}{a_1} \approx 0.25,$$

where $a_1$ and $a_2$ denote the usual phenomenological colour factors. The origin of the parameters $z$ and $y_{ew}$ is due to electroweak penguins. They are given by

$$z = \frac{|M_{ew}^C|}{|M_P|}, \quad y_{ew} = \frac{|M_{ew}^C|}{|M_{ew}|},$$

where $M_{ew}^C$ and $M_{ew}$ are the colour-suppressed and colour-allowed electroweak penguin amplitudes, again calculated by using generalized factorization. We have $z/(xy) \approx 0.75$ and $y_{ew} \approx y$, where the derivation of the numerical factor 0.75 can be found in [14]. Using
(35) and (36), it is an easy exercise to calculate \( \rho, T \) and \( P_{\text{ew}} \) in our simple model:

\[
\rho e^{i\theta} = -\frac{x(1 + y) \left(e^{i \Delta \phi} - 1\right)}{3 \frac{e^{i \phi_P}}{1 + (1 + 1/y_{\text{ew}}) \left(e^{i \Delta \phi} - 1\right)}} \tag{39}
\]

\[
|T| e^{i \delta_T} = x \left[1 + \frac{2}{3} \left(1 + y\right) \left(e^{i \Delta \phi} - 1\right)\right] |M_P| e^{i \phi_{1/2}} \tag{40}
\]

\[
|P_{\text{ew}}| e^{i \delta_{\text{ew}}} = z \left[1 + \frac{2}{3} \left(1 + \frac{1}{y_{\text{ew}}}\right) \left(e^{i \Delta \phi} - 1\right)\right] |M_P| e^{i \phi_{1/2}}, \tag{41}
\]

and correspondingly \( r \) and \( \epsilon \), which are obtained by normalizing \(|T|\) and \(|P_{\text{ew}}|\) through \(\sqrt{\langle P^2 \rangle}\) (see (13)).

The formulae given in [14] allow us to calculate all relevant observables of the \( B^\pm \to \pi^\pm K \) and \( B_d \to \pi^\pm K^\pm \) decays. In Fig. 1, we show the resulting dependence of the ratio \( R \) of combined \( B \to \pi K \) branching ratios (1) on the CKM angle \( \gamma \) for \( x = 0.2, y = 0.25, z = 0.0375, \phi_P = 180^\circ \) and various values of \( \Delta \phi \). In this figure, we have also included the curves corresponding to \( R_{\text{min}} \) (see (16)) in order to illustrate the way in which the corresponding bounds on \( \gamma \) are realized in this specific example. Note that the difference between the thin and thick solid lines is due to electroweak penguins.

Concerning the CP asymmetries \( A_0 \) and \( A_+ \), we “naturally” get values as large as \( \mathcal{O}(10\%) \). An interesting relation arises between these observables for \( \phi_P = 180^\circ \). If we neglect the electroweak penguin contributions for a moment, i.e. \( z = 0 \), we obtain

\[
A_0 = \frac{-6 x [(1 + y) \sin(\Delta \phi - \phi_P) - (2 - y) \sin \phi_P] \sin \gamma}{9 + 6 x (1 + y) \cos \phi_P - \cos(\Delta \phi - \phi_P) \cos \gamma + 2 x^2(1 + y)^2(1 - \cos \Delta \phi)} \tag{42}
\]

\[
A_+ = \frac{6 x (1 + y) [\sin(\Delta \phi - \phi_P) + \sin \phi_P] \sin \gamma}{9 + 6 x (1 + y) \cos \phi_P - \cos(\Delta \phi - \phi_P) \cos \gamma + 2 x^2(1 + y)^2(1 - \cos \Delta \phi)}. \tag{43}
\]

Consequently, in the case of \( \phi_P = 180^\circ \), we have \( A_0 = -A_+ \). This relation is only affected to a small extent by electroweak penguins. Since the CP asymmetries are, however, very sensitive to the strong phase \( \phi_P \), it may easily be spoiled through \( \phi_P \neq 180^\circ \).

At the end of the previous section we noted that the combined branching ratio \( \text{BR}(B^\pm \to K^\pm K) \) may be enhanced significantly through final-state interaction effects. This interesting feature can be seen nicely in Fig. 2, where we have used the same input parameters \( x, y, z \) and \( \phi_P \) as above, \( R_{SU(3)} = 0.7 \), the \( SU(3) \) relation (26), and \( \text{BR}(B^\pm \to \pi^\pm K) = 2.3 \times 10^{-5} \). The CP asymmetry \( A_+^{(d)} \), which depends strongly on the CKM angle \( \gamma \), may well be as large as \( \mathcal{O}(50\%) \).

Let us now have a closer look at the strategy to control the rescattering processes discussed in Section 3. To this end, we choose \( \gamma = 50^\circ, \Delta \phi = 45^\circ, \phi_P = 180^\circ \), and the same values for \( x, y \) and \( z \) as in our previous examples. Then we obtain \( R = 0.83, A_+ = -9.1\%, A_0 = 8.0\%, \text{BR}(B^\pm \to K^\pm K) = 7.9 \times 10^{-6} \) and \( A_+^{(d)} = 54\% \). For the \( B^\pm \to K^\pm K \) observables, we have assumed in addition \( \text{BR}(B^\pm \to \pi^\pm K) = 2.3 \times 10^{-5}, R_{SU(3)} = 0.7, \) and the \( SU(3) \) relation (26). Supposing that a future \( B \)-factory experiment
Figure 1: Illustration of the bounds on the CKM angle $\gamma$ arising from (16) within a simple model of final-state interactions specified in the text.
Figure 2: The dependence of the combined branching ratio $\text{BR}(B^\pm \to K^\pm K)$ on the CKM angle $\gamma$ for a simple model of final-state interactions specified in the text.
Figure 3: The dependence of $\rho$ determined with the help of (31) on the CKM angle $\gamma$ for a specific example discussed in the text.
Figure 4: Controlling rescattering effects in $R_{\text{min}}$ and the contours in the $\gamma$--$r$ plane through the $B^\pm \to K^\pm K$ observables for a specific example discussed in the text.
will find these values, the ratio (27) would imply $H = 3.3$ and $R_{SU(3)} = 0.7$. Using now (31), $\rho$ can be constrained as shown in Fig. 3, implying $\rho = 0.090 \pm 0.028$. The “true” value of $\rho$ is given in this example by 0.063 and is very close to its lower bound. The dot-dashed line in Fig. 3 corresponds to the “minimal” value $\rho_{\text{min}}$, which can be obtained from $A_+$ with the help of (19). The non-vanishing direct CP asymmetries also imply a range for $\gamma$, which is given by $32^\circ \leq \gamma \leq 148^\circ$, and excludes values around $0^\circ$ and $180^\circ$.

Using (31), the rescattering effects can be included in $R_{\text{min}}$ through (17). The corresponding curves are represented in Fig. 4 by the dot-dashed lines, whereas the dotted line corresponds to the “measured” value $R = 0.83$, which is larger than the present central value 0.65, and excludes the range $70^\circ \leq \gamma \leq 110^\circ$ around $90^\circ$. The contours in the $\gamma-r$ plane, taking into account the rescattering effects, are represented by the dashed lines in Fig. 4. As in Fig. 3, we have also included the curves corresponding to $\rho_{\text{min}}$ (the solid lines), which can be constructed by using only the $B \to \pi K$ observables, i.e. without making use of $B^{\pm} \to K^{\pm} K$. Consequently, in our example, the allowed range for $\gamma$ would be given by $32^\circ \leq \gamma \leq 70^\circ \vee 110^\circ \leq \gamma \leq 148^\circ$, while the “true” value is $\gamma = 50^\circ$. It is interesting to note that additional information on $r$ – in our example, the “true” value is 0.19 – would not lead to a significantly more stringent range for $\gamma$ in this case, as can be seen in Fig. 4.

Although electroweak penguins are included in our simple model and the values of the $B \to \pi K$ and $B^{\pm} \to K^{\pm} K$ observables calculated in this section, they are not included in the curves shown in Fig. 4. In the case of our example, we have $\epsilon = 0.089$ and $\Delta = 92^\circ$. Since $\Delta$ is very close to $90^\circ$, the electroweak penguin effects in the bound on $\gamma$ are only of second order in $\epsilon$, as can be seen in (17). Consequently, despite the large value of $\epsilon$ – the “short-distance” value is $\epsilon = \mathcal{O}(0.03)$ – electroweak penguins affect the constraints on $\gamma$ only to a small extent in our example. In general, however, electroweak penguin effects may represent the most important limitation of the theoretical accuracy of the bounds on $\gamma$. A detailed analysis can be found in [14].

### 5 Conclusions

We have shown that the decay $B^+ \to K^{\pm} \bar{K}^0$ and its charge conjugate allow us to take into account rescattering effects in constraints on the CKM angle $\gamma$ arising from $B \to \pi K$ modes. To accomplish this task, the $SU(3)$ flavour symmetry of strong interactions has to be used in order to relate $B^{\pm} \to K^{\pm} K$ to $B^{\pm} \to \pi^{\pm} K$. An important by-product of this approach is an allowed range for the parameter $\rho$, measuring the strength of the rescattering processes. Concerning the bounds on $\gamma$, $SU(3)$ breaking enters only at the “next-to-leading order” level, as it represents a correction to the correction that is due to the rescattering processes. Moreover, we have also indicated ways to explore the impact of $SU(3)$ breaking in a quantitative way.

In order to illustrate this strategy and the constraints on $\gamma$, we have used a simple model to describe final-state interactions. A realistic description is unfortunately out of reach at present, and would have to include, for instance, also inelastic rescattering...
contributions, which are expected to play an important role [21], and are neglected in this model. Other shortcomings are, for instance, the question of whether $a_1$ and $a_2$ take values similar to those measured in $B \to D^{(*)} \pi (\rho)$ decays, or the problem to fix the “short-distance” QCD penguin amplitude $|M_P|$. This model can therefore only serve to illustrate certain qualitative features of rescattering effects, for example their tendency to enhance the combined $B^\pm \to K^{\pm} K$ branching ratio significantly, or to induce sizeable CP violation in $B^\pm \to \pi^\pm K$.

In Refs. [2, 3], this model has been used to “demonstrate” that no useful constraints on the CKM angle $\gamma$ can be obtained from $B \to \pi K$ decays in the presence of final-state interactions (similar statements, although somewhat more moderate, have been made in [4, 5] within a different framework). Here we have shown – using exactly this model – that this is actually not the case. To this end, we have even chosen rather large values of $\Delta \phi$ in our examples, corresponding to large final-state interaction effects. In a recent attempt to calculate this strong phase by using a Regge pole model for $\pi K$ scattering, significantly smaller values, lying within the range between 14° and 20°, have been obtained [22]. In that case, there would not even be the need to correct at all for the final-state interaction effects in the bounds on $\gamma$. Certainly, future experimental data will tell us how important final-state interactions in $B \to \pi K$ decays really are.

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References


[21] See, for instance, the article by J. Donoghue et al. in [6].