New Bound on $\gamma$ from $B^{\pm} \to \pi K$ Decays

Matthias Neubert  
*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

and

Jonathan L. Rosner  
*Enrico Fermi Institute and Department of Physics*  
*University of Chicago, Chicago, IL 60637, USA*

**Abstract**

A new bound on the angle $\gamma$ of the unitarity triangle is derived using experimental information on the CP-averaged branching ratios for the rare decays $B^{\pm} \to \pi^{\pm}K^0$ and $B^{\pm} \to \pi^0K^{\pm}$. The theoretical description is cleaner than the Fleischer-Mannel analysis of the decays $B^{\pm} \to \pi^{\pm}K^0$ and $B^0 \to \pi^0K^{\pm}$ in that the two decay rates differ only in a single isospin amplitude, which has a simple structure in the SU(3) limit. As a consequence, electroweak penguin contributions and strong rescattering effects can be taken into account in a model-independent way. The resulting bound excludes values of $\cos \gamma$ around $0.6$ and is thus largely complementary to indirect constraints derived from a global analysis of the unitarity triangle.

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The study of CP violation in the weak decays of $B$ mesons is the main target of present and future $B$ factories. It will provide stringent tests of the flavor sector of the Standard Model and of the CKM paradigm, according to which all CP violation results from the presence of a single complex phase in the quark mixing matrix. The precise determination of the sides and angles of the unitarity triangle, which is a graphical representation of the unitarity relation $V_{ub}V_{us} + V_{cb}V_{cs} + V_{tb}V_{ts} = 0$, plays a central role in this program. Two of the angles of the triangle, $\beta$ and $\alpha$, will be accessible through measurements of CP violation in decays such as $B \to J/\psi K_S$ and $B \to \pi \pi$, though a model-independent analysis of the latter decays requires the difficult detection of the decay mode $B \to \pi^0 \pi^0$ [1]. The third angle, $\gamma$, is however harder to determine, making a direct experimental test of the triangle relation $\alpha + \beta + \gamma = 180^\circ$ a long-term objective.

Fleischer and Mannel have argued that some information on the angle $\gamma$ can be derived from the measurement of the branching ratios for the decays $B^\pm \to \pi^\pm K^0$ and $B^0 \to \pi^\pm K^\mp$, averaged over CP-conjugate modes [2]. In their original work, they obtained the bound

$$R = \frac{\tau(B^+) \ Br(B^0 \to \pi^- K^+) + Br(B^0 \to \pi^+ K^-)}{\tau(B^0) \ Br(B^+ \to \pi^+ K^0) + Br(B^- \to \pi^- K^0)} \geq \sin^2 \gamma,$$

(1)

which would exclude a region around $\gamma = 90^\circ$ provided that $R < 1$, a possibility allowed by the first measurement of this ratio yielding $R = 0.65 \pm 0.40$ [3]. It has later been realized, however, that the bound (1) may be subject to sizable corrections arising from final-state interactions and electroweak penguin contributions, which are difficult to quantify in a model-independent way [4]-[8]. To control such effects would require a more sophisticated analysis using information from other decays such as $B \to KK$ [9, 10]. In addition to these theoretical obstacles, the prospects for deriving useful information on $\gamma$ using the ratio $R$ are reduced by the fact that the CLEO Collaboration has announced a new measurement of this quantity yielding $R = 1.0 \pm 0.4$ [11].

In the present note we propose a variant of the Fleischer–Mannel analysis, which is theoretically cleaner and implies a highly non-trivial constraint on $\gamma$ provided present, preliminary data are confirmed by future measurements. Electroweak penguin contributions play an important role in this analysis, but they can be controlled in a model-independent way. Consider the ratio

$$R_s = \frac{Br(B^+ \to \pi^+ K^0) + Br(B^- \to \pi^- \bar{K}^0)}{2[Br(B^+ \to \pi^0 K^+) + Br(B^- \to \pi^0 K^-)]} \equiv (1 - \Delta_s)^2,$$

(2)

which would approach unity in the limit where $b \to s\bar{q}q$ penguin transitions involving top- or charm-quark loops dominate over the Cabibbo-suppressed $b \to u\bar{u}s$ transitions. Deviations from this limit are measured by the quantity $\Delta_s$. The CLEO Collaboration has recently reported the preliminary results $Br(B^\pm \to \pi^\pm K^0) = (1.4 \pm 0.5 \pm 0.2) \times 10^{-5}$ and $Br(B^\pm \to \pi^0 K^\pm) = (1.5 \pm 0.4 \pm 0.3) \times 10^{-5}$ for the CP-averaged branching ratios

\footnote{Some information on $\gamma$ can be obtained even when $R \approx 1$ if one uses separate measurements of the decay rates for $B^0 \to \pi^- K^+$ and $\bar{B}^0 \to \pi^+ K^-$ [4].}

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Experimental data as well as theoretical expectations indicate that the amplitude 
relevant to the further discussion are the isospin quantum numbers of these operators. 
From the isospin decomposition of the effective Hamiltonian it is obvious which operator 
corresponding to a deviation of larger than the amplitudes which includes the contributions of the QCD penguin operators, is an order of magnitude 
I differs from zero is responsible for the deviations of the ratios \( R \). The current–current operators \( Q_{1,2}^{\mu} \sim \bar{b}s\bar{u}u \) have components with \( \Delta I = 0 \) and \( \Delta I = 1 \); the current–current operators \( Q_{1,2}^{\mu} \sim \bar{b}s\bar{c}c \) and the QCD penguin operators \( Q_{3,6} \sim \bar{b}s \sum q\bar{q} \) have \( \Delta I = 0 \); the electroweak penguin operators \( Q_{1,6} \sim \bar{b}s \sum e_q\bar{q}q \), where \( e_q \) 
are the electric charges of the quarks, have \( \Delta I = 0 \) and \( \Delta I = 1 \). Since the initial \( B \) meson has \( I = \frac{3}{2} \) and the final states \((\pi K)\) can be decomposed into components with \( I = \frac{1}{2} \) and \( I = \frac{3}{2} \), the physical \( B \rightarrow \pi K \) decay amplitudes can be described in terms of three isospin amplitudes. They are called \( B_{1/2}, A_{1/2}, \) and \( A_{3/2} \) referring, respectively, to \( \Delta I = 0 \) with \( I_{\pi K} = \frac{1}{2} \), \( \Delta I = 1 \) with \( I_{\pi K} = \frac{1}{2} \), and \( \Delta I = 1 \) with \( I_{\pi K} = \frac{3}{2} \) [6, 13, 14]. From the isospin decomposition of the effective Hamiltonian it is obvious which operator matrix elements and weak phases enter the various isospin amplitudes. The resulting expressions for the decay amplitudes relevant to our discussion are 
\[
\mathcal{A}(B^+ \rightarrow \pi^+ K^0) = B_{1/2} + A_{1/2} + A_{3/2}, \\
-\sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+) = B_{1/2} + A_{1/2} - 2A_{3/2}, \\
-\mathcal{A}(B^0 \rightarrow \pi^- K^+) = B_{1/2} - A_{1/2} - A_{3/2}.
\]
Experimental data as well as theoretical expectations indicate that the amplitude \( B_{1/2} \), which includes the contributions of the QCD penguin operators, is an order of magnitude larger than the amplitudes \( A_{1/2} \) and \( A_{3/2} \) [6, 15]. Yet, the fact that \( A_{1/2} \) and \( A_{3/2} \) are different from zero is responsible for the deviations of the ratios \( R \). An important advantage of our analysis is that the two decay amplitudes entering the ratio \( R \) in (2) differ only in the sign of the isospin amplitude \( A_{3/2} \), which in the SU(3) limit is related to a matrix element of a single combination of local four-quark operators in the effective weak Hamiltonian. Despite the presence of contributions carrying different weak phases, there is thus a single strong-interaction phase associated with this amplitude. This fact will allow us to control rescattering effects and calculate electroweak penguin contributions in a model-independent way. On the contrary, the decay amplitudes entering the ratio \( R \) considered in the original Fleischer–Mannel analysis differ in
the signs of the two isospin amplitudes $A_{1/2}$ and $A_{3/2}$, introducing hadronic uncertainties into the calculation [4–8].

Taking advantage of the fact that the top- and charm-quark penguin contributions to $B_{1/2}$ are much larger than all other contributions to the isospin amplitudes, we write $B_{1/2} = e^{i\phi} e^{i\phi_{P}} |P_{d}| + O(\varepsilon)$, where $e^{i\phi_{P}}$ is a strong-interaction phase, and $e^{i\phi}$ is the weak phase associated with the top-quark penguin contribution. The parameter $\varepsilon \ll 1$ controls the size of the Cabibbo-suppressed $b \to u\bar{n}s$ transitions relative to the leading penguin amplitudes. The main result of the present letter is that the isospin amplitude $A_{3/2}$, which causes the deviation of the ratio $R_{s}$ from unity, can be written as

$$\frac{3A_{3/2}}{|P_{d}|} = -\varepsilon e^{i\phi_{3/2}}(e^{i\gamma} - \delta_{\text{EW}}), \quad (6)$$

where $e^{i\phi_{3/2}}$ is a strong-interaction phase, $e^{i\gamma}$ is the weak phase of the parameter $\lambda_{u}$ associated with the $b \to u\bar{n}s$ transitions, and $\delta_{\text{EW}}$ is to a good approximation a real parameter accounting for the contributions of electroweak penguin operators. The fact that this parameter does not carry a non-trivial strong-interaction phase is crucial and will be derived below. We define the parameter $\varepsilon$ to be real and positive. In the diagrammatic amplitude approach of flavor-flow topologies, $\varepsilon = |T + C|/|P_{d}|$ is the ratio of color-allowed plus color-suppressed tree amplitudes to the sum of the top- and charm-quark penguin amplitudes [16]–[18]. Using the parametrization (6), we obtain for the quantity $\Delta_{s}$ defined in (2) the result

$$\Delta_{s} = -\varepsilon \cos \Delta \phi (\cos \gamma - \delta_{\text{EW}}) + O(\varepsilon^{2}), \quad (7)$$

where $\Delta \phi = \phi_{3/2} - \phi_{P}$. The phase conventions adopted above take into account the naive phase relation between penguin and tree contributions, such that all non-trivial strong-interaction phases are accounted for by the phase difference $\Delta \phi$.

The value of the parameter $\varepsilon$ can be estimated using various experimental and theoretical information. The penguin contribution $|P_{d}| \approx |B_{1/2}|$ can be extracted from the branching ratio for the decays $B^{\pm} \to \pi^{\pm} K^{0}$, which are expected to receive only a very small contamination from other contributions such as up-quark penguins or annihilation diagrams. Using the experimental value for this branching ratio quoted earlier yields $|P_{d}| = (3.74 \pm 0.72) \times 10^{-3}$. We give amplitudes in “branching ratio units”, so that $\text{Br} = |A|^{2}$. The tree contribution $|T + C|$ can be estimated experimentally invoking SU(3) symmetry, in which case it can be extracted from the branching ratio for the decay $B^{\pm} \to \pi^{\pm} \pi^{0}$ using the relation [17]–[19]

$$T + C = -\sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_{K}}{f_{\pi}} A(B^{+} \to \pi^{+} \pi^{0}), \quad (8)$$

where the factor $f_{K}/f_{\pi} = 1.22 \pm 0.01$ accounts for the leading (i.e., factorizable) SU(3)-breaking corrections. The CLEO Collaboration has recently reported a signal for this decay mode with a significance of 2.3 standard deviations. They quote the upper limit $\text{Br}(B^{\pm} \to \pi^{\pm} \pi^{0}) < 1.6 \times 10^{-6}$ at 90% confidence level [11], which implies $|T + C| <
1.56 \times 10^{-3}. Taking this signal seriously, we may derive from the reported event rate and efficiency the branching ratio \( \text{Br}(B^\pm \to \pi^\pm \pi^0) = (0.59^{+0.22}_{-0.21}) \times 10^{-3} \), which yields the value \( |T + C| = (0.94^{+0.25}_{-0.21}) \times 10^{-3} \). The tree contribution may also be obtained theoretically employing the factorization hypothesis. Using the values of the hadronic form factors determined in Ref. [20] combined with the new world average \( |V_{ub}| = (3.56 \pm 0.56) \times 10^{-3} \) [21], we get \( |T + C| = (0.88 \pm 0.23) \times 10^{-3} \), in good agreement with the estimate obtained using experimental data. We take the average of the two numbers and combine it with the experimentally determined value for the penguin amplitude to obtain \( \varepsilon = 0.24 \pm 0.06 \). It is important that, once the branching ratio for the decays \( B^\pm \to \pi^\pm \pi^0 \) and \( B^\pm \to \pi^\pm K^0 \) are measured more precisely, the parameter \( \varepsilon \) can be determined with smaller errors and without almost any (i.e., up to non-factorizable SU(3)-breaking corrections) recourse to theory. Note that with the above value of \( \varepsilon \) the terms of \( O(\varepsilon^2) \) omitted in (7) are expected to be reasonably small in view of the sizable experimental uncertainties in (3). Nevertheless, we shall comment on the relevance of such higher-order terms below.

With the above value for \( \varepsilon \), the relation (7) implies

\[
\cos \Delta \phi (\delta_{\text{EW}} - \cos \gamma) = \frac{\Delta_4}{\varepsilon} = 1.33 \pm 0.78
\]

\[
\Rightarrow \ |\delta_{\text{EW}} - \cos \gamma| \geq 1.33 \pm 0.78 .
\]  

(9)

The last step missing to turn this result into a useful constraint on \( \gamma \) is an estimate of the contribution from electroweak penguins to the isospin amplitude \( A_{3/2} \). This can be done without encountering hadronic uncertainties. The crucial observation is that the electroweak penguin operators \( Q_9 \) and \( Q_{10} \), whose Wilson coefficients are enhanced by the large mass of the top quark, are Fierz-equivalent to the current-current operators \( Q_1 \) and \( Q_2 \). As a result, the \( \Delta I = 1 \) part of the effective weak Hamiltonian for \( B \to \pi K \) decays can be written as [6, 9]

\[
\mathcal{H}_{\Delta I = 1} = \frac{G_F}{\sqrt{2}} \left\{ \left( \lambda_u C_1 - \frac{3}{2} \lambda_t C_9 \right) \bar{Q}_1 + \left( \lambda_u C_2 - \frac{3}{2} \lambda_t C_{10} \right) \bar{Q}_2 + \ldots \right\} + \text{h.c.},
\]  

(10)

where \( \bar{Q}_i = \frac{1}{2}(Q_i^\mu - Q_i^\tau) \). The dots represent the contributions from the electroweak penguin operators \( Q_7 \) and \( Q_8 \), which have a different Dirac structure. The Wilson coefficients of these operators are so small that their contributions can be safely neglected. It is important in this context that for heavy mesons the matrix elements of four-quark operators with Dirac structure \( (V - A) \otimes (V + A) \) are not enhanced with respect to those of operators with the usual \( (V - A) \otimes (V - A) \) structure, as can be checked explicitly in the factorization approximation. To an excellent approximation, the net effect of electroweak penguin contributions to the \( \Delta I = 1 \) isospin amplitudes in \( B \to \pi K \) decays thus consists of the replacements of the Wilson coefficients \( C_1 \) and \( C_2 \) of the current-current operators with the combinations shown in (10). In the SU(3) limit, \( U \)-spin invariance implies that the isospin amplitude \( A_{3/2} \) is given by the matrix element of the combination \( (\bar{Q}_1 + \bar{Q}_2) \) only [19], the coefficient of which is \( \frac{1}{2} \lambda_t [C_1 + C_2 - \lambda_t (C_9 + C_{10})] \). The difference, \( (\bar{Q}_1 - \bar{Q}_2) \), does not contribute to this amplitude. To see this, consider the relation (8)
between the current–current part of the amplitude $A_{3/2}$ and the amplitude for the decay $B^+ \to \pi^+\pi^0$. Bose symmetry requires the two pions to be in an $I = 2$ state, so that only the $\Delta I = \frac{3}{2}$ part of the effective weak Hamiltonian for $B \to \pi\pi$ decays contributes to the decay. Since the combination $(Q_1' - Q_2')$, where the prime on the operators indicates the replacement $s \to d$ appropriate for $\Delta S = 0$ transitions, has $\Delta I = \frac{1}{2}$, the decay $B^+ \to \pi^+\pi^0$ receives a contribution from the combination $(Q_1' + Q_2')$ only. SU(3) symmetry then implies that it is only the combination $(Q_1 + Q_2)$ which contributes to the isospin amplitude $A_{3/2}$ in $B \to \pi K$ decays. It follows that the quantity $\delta_{\text{EW}}$ in (6) is given by

$$\delta_{\text{EW}} = -\frac{3}{2\lambda^2 R_b} \frac{C_9 + C_{10}}{C_1 + C_2} = 0.66 \pm 0.11,$$

where we have used $\lambda_u/\lambda_t \approx -\lambda^2 R_b e^{\gamma}$ with $\lambda \approx 0.22$ and $R_b = \lambda^{-1} |V_{ub}/V_{cb}| \approx 0.41 \pm 0.07$. The ratio of the Wilson coefficients is to a very good approximation independent of the choice of the renormalization scale. We take $(C_9 + C_{10})/(C_1 + C_2) = -1.14\alpha$, which is obtained using the leading-order coefficients\(^2\) at the scale $\mu = m_b$ [12]. For the electromagnetic coupling at the weak scale we take $\alpha = 1/129$. Note that the main uncertainty in the value of $\delta_{\text{EW}}$ results from the uncertainty in $V_{ub}$, which is likely to be reduced in the near future.

SU(3)-breaking corrections to the relation (11) are controlled by the ratio of the following operator matrix elements:

$$\frac{\langle \pi K(I = \frac{3}{2}) | Q_1 - Q_2 | B^+ \rangle}{\langle \pi K(I = \frac{3}{2}) | Q_1 + Q_2 | B^+ \rangle} \equiv -\delta_{\text{SU}(3)} e^{i\Delta \varphi}.$$

The magnitude of this ratio can be estimated using the generalized factorization hypothesis [20] to calculate the matrix elements of the current–current operators. This leads to

$$\delta_{\text{SU}(3)} = \frac{1 - \zeta a_K - a_\pi}{1 + \zeta a_K + a_\pi} \approx 1-3\%, \quad \Delta \varphi = 0,$$

where $a_K = f_K(m_B^2 - m_K^2)F_0^{B\to\pi}(m_K^2)$ and $a_\pi = f_\pi(m_B^2 - m_B^2)F_0^{B\to\pi}(m_B^2)$ are combinations of hadronic matrix elements, and $\zeta \approx 0.45$ is a parameter controlling non-factorizable corrections. Despite the fact that the factorization approximation may be crude, this estimate suggests that SU(3)-breaking corrections to the result (11) are very small. To linear order in SU(3) breaking, the results (7) and (9) can be corrected by replacing $\delta_{\text{EW}}$ with the effective value

$$\delta_{\text{EW}}^{\text{eff}} = \left(1 - k \delta_{\text{SU}(3)} \frac{\cos(\Delta \phi + \Delta \varphi)}{\cos \Delta \phi}\right) \delta_{\text{EW}},$$

where

$$k = \frac{C_2 - C_1}{C_2 + C_1} + \frac{C_9 - C_{10}}{C_9 + C_{10}} \approx 3.43$$

\(^2\)Using next-to-leading order coefficients would change this value by less than 2%; however, at this order the coefficient functions may depend on the choice of the operator basis, so one has to be careful when using Fierz identities.
Table 1: Constraints on $\gamma$ for different values of the ratio $\Delta_s/\varepsilon$

<table>
<thead>
<tr>
<th>$\Delta_s/\varepsilon$</th>
<th>deviation from central value</th>
<th>bound on $\cos\gamma$</th>
<th>excluded region</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.11</td>
<td>$+1\sigma$</td>
<td>no solution</td>
<td>all</td>
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<tr>
<td>1.33</td>
<td>central value</td>
<td>$&lt; -0.55$</td>
<td>$</td>
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<tr>
<td>0.55</td>
<td>$-1\sigma$</td>
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<td>$</td>
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<tr>
<td>0.16</td>
<td>$-1.5\sigma$</td>
<td>$&gt; 0.64$ or $&lt; 0.62$</td>
<td>$50^\circ &lt;</td>
</tr>
</tbody>
</table>

is a combination of Wilson coefficients. Since $(\bar{Q}_1 - \bar{Q}_2)$ and $(\bar{Q}_1 + \bar{Q}_2)$ are local operators, it is unlikely that the strong-interaction phase shift $\Delta\varphi$ could acquire any significant value. We thus expect that the main effect of SU(3) breaking is to reduce the value of $\delta_{\text{EW}}$ in (11) by an amount of order 5%. To account for this effect, we shall from now on use the value $\delta_{\text{EW}} = 0.63 \pm 0.15$ with an increased error, which is large enough to cover possible small contributions from a non-zero phase-shift $\Delta\varphi$ or deviations from the factorization approximation. It is interesting to note that our general results for the structure of the electroweak penguin contributions to the isospin amplitude $A_{3/2}$, including the pattern of SU(3)-breaking effects, are in full accord with a model estimate by Deshpande and He [22]. However, these authors did not realize that such contributions can be calculated in an essentially model-independent way. As a result, electroweak penguin effects can be taken into account in the method proposed in Ref. [17] for learning $\gamma$ by comparing the rates for the processes $B^+ \rightarrow \pi^+ K^0$, $\pi^0 K^+$, and $\pi^+ \pi^0$ with their charge conjugates. This will be discussed in more detail elsewhere.

The role of electroweak penguins in the bound (9) is an important one, and it is crucial that the parameter $\delta_{\text{EW}}$ can be calculated in a reliable way in terms of perturbative Wilson coefficients and measured quantities. The result is that the constraint (9) excludes values of $|\gamma|$ around $\arccos(0.63) \approx 51^\circ$. More precisely, the excluded region depends on the value of the experimental number for the ratio $\Delta_s/\varepsilon$ on the right-hand side of the bound. The allowed regions for $\cos\gamma$ must satisfy $\cos\gamma > \delta_{\text{EW}} + \Delta_s/\varepsilon$ or $\cos\gamma < \delta_{\text{EW}} - \Delta_s/\varepsilon$. In evaluating these results we lower or increase the value of $\delta_{\text{EW}}$ by one standard deviation so as to be conservative, i.e., we use $\delta_{\text{EW}} = 0.48$ in the first relation and $\delta_{\text{EW}} = 0.78$ in the second one. The first solution only exists if $\Delta_s/\varepsilon < 1 - \delta_{\text{EW}}$. If $\Delta_s/\varepsilon > 1 + \delta_{\text{EW}}$ there is no solution for $\gamma$ at all. The results obtained for some representative values of $\Delta_s/\varepsilon$ are shown in Table 1. It is remarkable that the constraints on $\gamma$ derived from the bound (9) are to a large extent complementary to the preferred region for this angle obtained from a global analysis of the unitarity triangle, using information from semileptonic $B$ decays, $B^-\bar{B}$ mixing, and CP violation in the kaon system. A typical range of values allowed by such an analysis is $47^\circ < \gamma < 105^\circ$ [23], where the upper bound is determined by the improved lower limit on the quantity $\Delta m_s$ controlling $B^-\bar{B}$ mixing [11]. For the central value of $\Delta_s/\varepsilon$ our bound is inconsistent with this region, while for the 1$\sigma$ lower value $\Delta_s/\varepsilon = 0.55$ it would exclude values of $|\gamma|$
below 77°, thus leaving the rather narrow range 77° < \gamma < 105°. Clearly, a more precise measurement of the parameter \Delta_s/\varepsilon could potentially yield a very non-trivial constraint on \gamma and provide a stringent test of the CKM paradigm.

Potentially the weakest point of our analysis appears to be the neglect of higher-order terms in the ratio \varepsilon in the expression for the quantity \Delta_s in (7), given that \varepsilon \approx 0.24 is not such a small expansion parameter. We recall that \varepsilon is a measure of the strength of the Cabibbo-suppressed \( b \to s\bar{u}u \) transitions, which carry the weak phase \( e^{i\gamma} \), relative to the leading penguin transitions. Whereas a full account of all higher-order terms of this kind would introduce unknown strong-interaction parameters, we now show that the potentially most important terms can be analysed without much additional complication. The reason is that contributions proportional to the weak phase \( e^{i\gamma} \) in the decay amplitude for \( B^+ \to \pi^+K^0 \) are likely to be much smaller than those in the amplitude for \( B^+ \to \pi^0K^+ \), because they could only be induced via final-state rescattering through soft annihilation or up-quark penguin topologies [6]. Model estimates indicate that those effects typically lead to contributions of order a few percent \([4]-[8]\). This observation justifies treating these contributions from a numerical point of view as quadratic rather than linear in \varepsilon, in which case they enter the theoretical result for the quantity \Delta_s only at the negligible level of \( O(\varepsilon^3) \). The generalization of the relation (7) then becomes

\[
\Delta_s = \varepsilon \cos \Delta \phi \left( \delta_{\text{EW}} - \cos \gamma \right) - \frac{\varepsilon^2}{2} \left[ (3 \cos^2 \Delta \phi - 1)(\delta_{\text{EW}} - \cos \gamma)^2 - \sin^2 \gamma \right] + O(\varepsilon^3) .
\]

(16)

The question is whether the terms of order \( \varepsilon^2 \) could increase the value of \Delta_s, thereby weakening the bound on \gamma derived in this letter. Since \varepsilon \ll 1, the quantity \Delta_s takes its maximum value for \( \left| \cos \Delta \phi \right| \approx 1 \), so that

\[
\Delta_s \leq \varepsilon \left| \delta_{\text{EW}} - \cos \gamma \right| - \left( \frac{\Delta_s^2}{2} - \frac{\varepsilon^2}{2} \sin^2 \gamma \right) + O(\varepsilon^3) .
\]

(17)

If experimentally it is found that \( \Delta_s > \varepsilon/\sqrt{2} \approx 0.17 \), the second term is negative and strengthens the bound on \gamma. Provided future measurements confirm the present value of \Delta_s in (3) within one standard deviation, the bound (9) derived in linear approximation is thus a conservative result. On the other hand, if \Delta_s would turn out to be much smaller than the current central value, the bound may be weakened by higher-order terms, but those would generally be smaller than \( \varepsilon^2/2 \approx 3\% \), which is an almost negligible amount.

In summary, we have shown that a non-trivial bound on the angle \gamma of the unitarity triangle can be derived from a measurement of the branching ratios for the decays \( B^{\pm} \to \pi K \), averaged over CP-conjugate modes. Electroweak penguin contributions, which play an important role in these decays, can be controlled in a model-independent way using Fierz identities and SU(3) symmetry relations, with strong indications that SU(3)-breaking corrections are very small.

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[22] N.G. Deshpande and X.-G. He, Phys. Rev. Lett. 74 (1995) 26 [E: 74 (1995) 4099]. See, in particular, their eq. (17), implying $\delta_{EW} \approx 0.80$ once we recognize the absence of a non-trivial final-state relative phase between the $I = 3/2$ tree and electroweak penguin terms. The remaining difference between this result and ours is due to different values used for the CKM parameters and Wilson coefficients.