HEAVY QUARKS FROM QCD SPECTRAL SUM RULES

Abstract

This is a short review of the present status of the dynamics of the heavy quarks extracted from QCD spectral sum rules. We mention shortly some properties of the hybrid and B mesons, the value of $V_{cb}$, the determination of the "perturbative" quark mass, the decay constants of the $D$ and $B$ mesons, the slope of the $B$-meson $W$-function, and the extracted parameters of the Isgur-Wise function. We discuss the determination of the "perturbative" quark mass from QCD spectral sum rules. We mention briefly some properties of the hybrid and $B_c$ mesons.

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1 Introduction

We have been living with QCD spectral sum rules (QSSR) (or QCD sum rules for ITEP sum rules for hadronic sum rules...) for 15 years within the impressive ability of the method for describing the complex phenomena of hadronic physics with the few universal “fundamental” parameters of the QCD Lagrangian (QCD coupling $\alpha_s$, quark masses and vacuum condensates built from the quarks and/or gluon fields which parametrize the non-perturbative phenomena). The approach might be very close to the lattice calculations as it also uses the first principles of QCD but unlike the case of the lattice which is based on sophisticated numerical simulations QSSR is quite simple as it is a semi-analytic approach based on a semiperturbative expansion and Feynmann graph techniques implemented in an Operator Product Expansion (OPE) where the condensates contribute as higher dimension operators in the OPE. In this approach one can really control and in some sense understand the origin of the numbers obtained from the analysis.

Since its early days and up to now the field has unfortunately suffered from some apparent controversy and (worse) from some irrational emotional fights between different groups which obscure its beauty. But when you start d’y fourrer votre nez, you realize that the QCD spectral sum rule is one of the most interesting discoveries of the last decade since with its simplicity it can describe in an elegant way the complexity of the hadron phenomena without waiting for a complete understanding of the confinement problem.

One can fairly say that QCD spectral sum rules already started before QCD in the time of current algebra in 60s when peoples proposed different ad hoc superconvergence sum rules especially the Weinberg and Das–Mathur–Okubo sum rules which came under control only within the advent of QCD [1]. However the main flow comes from the classic paper of Shifman–Vainshtein–Zakharov [2] (hereafter referred to as SVZ) which goes beyond the naïve perturbation theory thanks to the inclusion of the vacuum condensate effects in the OPE. More details and more complete discussions of QSSR and its various applications to hadron physics can be found for instance in [3].

In this talk I shall present aspects of QSSR in the analysis of the properties of heavy flavours. As I am limited in space-time (an extended and updated version of this talk will be published in the Proceedings of QCD 94 at Montpellier) I cannot cover in detail all QSSR applications to the heavy-quark physics here. I will only shortly discuss the following topics which I think are important in the development of the understanding of the heavy-quark properties. These concern the determination of the:

- heavy-quark “perturbative” pole and running masses
- decay constants of the $D_1^0 B$ and $D_s \Gamma B_s$ mesons and the $B_B$-parameter
- semileptonic and radiative decay form factors of the $B$-mesons
- slope of the Isgur–Wise function and the emerged value of $V_{cb}$
- properties of hybrids and $B_s$ mesons.
2 QCD spectral sum rules

In order to illustrate the QSSR method in a pedagogical way let us consider the two-point correlator:

$$
\Pi_\mu^\nu(q^2, M_b^2) \equiv i \int d^4x \ e^{ixq} \langle 0 | TJ_\mu^\nu(x)(J_\nu^\mu(0)) | 0 \rangle \tag{1}
$$

where $J_\mu^\nu(x) \equiv \bar{b}\gamma^\mu b(x)$ is the local vector current of the $b-$quark. The correlator obeys the well-known Källen-Lehmann dispersion relation:

$$
\Pi_b(q^2, M_b^2) = \int_{AM_b^2}^{\infty} \frac{dt}{t - q^2 - ie} \frac{1}{\pi} \text{Im} \Pi_b(t) + \text{subtractions}, \tag{2}
$$

which expresses in a clear way the duality between the spectral function $\text{Im} \Pi_b(t)$ which can be measured experimentally as here it is related to the $e^+e^-$ into $\Upsilon$-like states total cross-section while $\Pi_b(q^2, M_b^2)$ can be calculated directly in QCD even at $q^2 = 0$ thanks to the fact that $M_b^2 - q^2 \gg \Lambda^2$. The QSSR is an improvement on the previous dispersion relation.

In the QCD side such an improvement is achieved by adding to the usual perturbative expression of the correlator the non-perturbative contributions as parametrized by the vacuum condensates of higher and higher dimensions in the OPE [2]:

$$
\Pi_b(q^2, M_b^2) \sim \sum_{D=0,2,4, \ldots} \frac{1}{(M_b^2 - q^2)^{D/2}} \sum_{\text{dim}O = D} C^{(J)}(q^2, M_b^2, \mu) \langle O(\mu) \rangle, \tag{3}
$$

where $\mu$ is an arbitrary scale that separates the long- and short-distance dynamics; $C^{(J)}$ are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques: $D = 0$ corresponds to the case of naïve perturbative contribution; $\langle O \rangle$ are the non-perturbative condensates built from the quarks or/and gluon fields. For $D = 4$ the condensates that can be formed are the quark $M_i(\bar{\psi}\psi)$ and gluon $\langle \alpha_s G^2 \rangle$ ones; for $D = 5$ one can have the mixed quark-gluon condensate $\langle \bar{\psi} \sigma_{\mu\nu} \lambda^a / 2G^a_{\mu\nu} \psi \rangle$ while for $D = 6$ one has for instance the triple gluon $gf_{abc} \langle G^a G^b G^c \rangle$ and the four-quark $\alpha_s \langle \bar{\psi}_1 \gamma_5 \psi_1 \Gamma_2 \psi_2 \rangle$ where $\Gamma_i$ are generic notations for any Dirac and colour matrices. The validity of this expansion has been understood formally using renormalon techniques [4] and by building renormalization invariant combinations of the condensates (Appendix of [5]). The SVZ-expansion is phenomenologically confirmed from the unexpected accurate determination of the QCD coupling $\alpha_s$ from semi-inclusive tau decays [5,6]. In the present case of heavy-heavy correlator the OPE is much simpler as one can show [7,8] that the heavy-quark condensate effects can be included into those of the gluon condensates such that up to $D \leq 6$ only the $C^2$ and $G^2$ condensates appear in the OPE. Indeed the SVZ has originally exploited this feature for their first estimate of the gluon condensate value.

For the phenomenological side the improvement comes from the use of either finite number of derivatives and finite values of $q^2$ (moment sum rules):

$$
\mathcal{M}^{(n)} \equiv \frac{1}{n!} \left. \frac{\partial^n \Pi_b(q^2)}{(\partial q^2)^n} \right|_{q^2 = 0} = \int_{AM_b^2}^{\infty} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im} \Pi_b(t), \tag{4}
$$

For $n = 1$ this reads:

$$
\mathcal{M}^{(1)} = \int_{AM_b^2}^{\infty} \frac{dt}{t^2} \frac{1}{\pi} \text{Im} \Pi_b(t) \tag{5}
$$

where $\mathcal{M}^{(1)}$ is called the heavy quark moments.
or infinite number of derivatives and infinite values of \( q^2 \) but keeping their ratio fixed as 
\[
\tau \equiv n/q^2 \quad \text{(Laplace or Borel or exponential sum rules)}:
\]
\[
\mathcal{L}(\tau, M^2_t) = \int_{4M_t^2}^{\infty} dt \exp(-t\tau) \frac{1}{\pi} \text{Im}\Pi_t(t).
\]
There also exists non-relativistic versions of these two sum rules which are convenient quantities to work with in the large-quark-mass limit. In these cases one introduces non-relativistic variables \( E \) and \( \tau_N \):
\[
 t \equiv (E + M_b)^2 \quad \text{and} \quad \tau_N = 4M_b\tau.
\]
In the previous sum rules the gain comes from the weight factors which enhance the contribution of the lowest ground state meson to the spectral integral. This fact makes the simple duality ansatz parametrization:
\[
\text{“one resonance”} \delta(t - M_R^2) + \text{“QCD continuum”} \Theta(t - t_c),
\]
of the spectral function to give a very good description of the spectral integral where the resonance enters via its coupling to the quark current. In the case of the \( \Upsilon \Gamma \) this coupling can be defined as:
\[
\langle 0|\bar{b}\gamma^\mu b|\Upsilon \rangle = \sqrt{2}M_s^4\frac{M_b^2}{2\gamma_\gamma}.
\]
The previous feature has been tested in the light-quark channel from the \( e^+e^- \rightarrow 1=1 \) hadron data and in the heavy-quark ones from the \( e^+e^- \rightarrow \Upsilon \) or \( \psi \) data within a good accuracy. To the previous sum rules one can also add the ratios:
\[
\mathcal{R}^{(n)} \equiv \frac{M^{(n)}}{M^{(n+1)}} \quad \text{and} \quad \mathcal{R}_t \equiv -\frac{d}{d\tau} \log \mathcal{L},
\]
and their finite energy sum rule (FESR) variants in order to fix the squared mass of the ground state. In principle the pairs \( (n, t_c) \) are free external parameters in the analysis so that the optimal result should be insensitive to their variations. Stability criteria which are equivalent to the variational method state that the optimal results should be obtained at the minimas or at the inflexion points in \( n \) or \( \tau \) while stability in \( t_c \) is useful to control the sensitivity of the result in the changes of \( t_c \)-values. To these stability criteria are added constraints from local duality FESR which correlate the \( t_c \)-value to those of the ground state mass and coupling \([9]\). Stability criteria have also been tested in models such as harmonic oscillator where the exact and approximate solutions are known \([10]\). The most conservative optimization criteria which include various types of optimizations in the literature are the following: the optimal result is obtained in the region from the beginning of \( \tau/n \) stability (this corresponds in most of the cases to the so-called plateau discussed often in the literature but to my opinion the interpretation of this nice plateau as a good sign of a good continuum model is not sufficient because the flatness of the curve extends in the uninteresting high-energy region where the properties of the ground state are lost) until the beginning of the \( t_c \) stability where the value of \( t_c \) corresponds to about the one fixed by FESR duality constraints. The earlier sum rule window introduced by SVZ stating that the optimal result should be in the region where both the non-perturbative and continuum contributions are small is included in the previous region. Indeed at the stability point we have an equilibrium between the continuum and non-perturbative contributions which are both small while the OPE is still convergent at this point.
3 The “perturbative” masses

We shall only discuss the determinations of the quark masses from relativistic sum rules which use a truncated perturbative series where the perturbative masses entering in the sum rule analysis are then well-defined. This is not the case of non-relativistic sum rules and heavy-quark effective theory where the quark is considered to be infinitely massive and the QCD series is summed up. In this case the pole mass is ill-defined due to the parasitic renormalon effects which induce after a resummation of the QCD series at large order a “non-perturbative” piece of the order of $\Lambda$ (however this effect is regularization-scheme dependent and is absent in the $\overline{MS}$-scheme) [11]. Also in this infinite mass limit the effects of the coulombic terms are important which render the estimate from non-relativistic sum rules very sensitive to the value of $\alpha_s$ used in the analysis. These non-perturbative effects induce a kind of dressed quark where the associated ill-defined pole mass has a strength similar to the one used in potential models but higher than the one obtained from the standard relativistic sum rule at finite values of the quark mass and for a truncated QCD series. In the same way non-perturbative effects due to hadronization shift to higher values the mass obtained from a fit of the inclusive decays and deep inelastic scattering data.

The masses obtained from the relativistic sum rules should be identified like in the case of the light quarks with the so-called current mass entering in the QCD Lagrangian which is a well-defined quantity in perturbation theory within the $\overline{MS}$-scheme. Using relativistic sum rules in the $\psi$ and $\Upsilon$ channels different groups have obtained the Euclidean mass [3]:

$$M_c(p^2 = -M_c^2) \simeq (1.26 \pm 0.02) \text{ GeV} \quad M_b(p^2 = -M_b^2) \simeq (4.18 \pm 0.02) \text{ GeV},$$  \hspace{1cm} (10)

defined in the Landau gauge\(^1\). However the use of this gauge-dependent mass is not inconvenient as we know that the correlator entering the sum rule analysis is gauge-invariant. Instead the use of the Euclidean mass minimizes the size of the radiative corrections at the order where the sum rule analysis is done making the result insensitive to the error in the value of $\alpha_s$. One can translate this result into the perturbative pole mass through the relation:

$$M_Q(p^2 = M_Q^2) \simeq M_Q(p^2 = -M_Q^2) \left(1 + 2 \log 2 \frac{\alpha_s}{\pi}\right),$$  \hspace{1cm} (11)

from which one can deduce the value of the perturbative pole masses which are gauge-invariant:

$$M_c(p^2 = M_c^2) \simeq (1.46 \pm 0.05) \text{ GeV} \quad M_b(p^2 = M_b^2) \simeq (4.64 \pm 0.06) \text{ GeV},$$  \hspace{1cm} (12)

for $\Lambda_5 \simeq (180 \pm 80) \text{ MeV}$. One can also estimate the perturbative $b$-pole mass by using the ratios of the relativistic sum rules in the $B$ and $B^*$ channels. One obtains [3]:

$$M_b(p^2 = M_b^2) \simeq (4.56 \pm 0.05) \text{ GeV},$$  \hspace{1cm} (13)

\(^1\)We have included in the average the slightly lower and more precise value $M_b(p^2 = -M_b^2) = (4.17 \pm 0.02) \text{ GeV}$ [12] obtained in this channel from relativistic moment sum rules.
By taking the average of their values from the previous two independent sources one can deduce (see also [13]):

\[ M_b(p^2 = M_b^2) \simeq (4.59 \pm 0.04) \text{ GeV}, \]  

(14)

The value of the perturbative \( b \)-pole mass is definitely lower than the ones in the range 4.8-5.3 GeV from non-relativistic approaches and inclusive data due to the presence of the non-perturbative components induced by the summation of the QCD series in these latter cases. The running masses in the \( \overline{MS} \)-scheme can be deduced directly from the Euclidian masses in (10) or equivalently from the perturbative pole masses in (12) and (13): through (derived for the first time in [13]):

\[ M_Q(p^2 = M_Q^2) \simeq M_Q(M_Q^2) \left( 1 + \frac{4 \alpha_s}{3 \pi} \right), \]  

(15)

One obtains:

\[ M_c(1 \text{ GeV}) \simeq (1.40 \pm 0.06) \text{ GeV} \quad M_b(1 \text{ GeV}) \simeq (5.87 \pm 0.06) \text{ GeV}, \]  

(16)

which, combined with the \( s \)-quark mass values in [13], gives:

\[ M_b/m_s \simeq 36.7 \pm 2.2, \]  

(17)

a result of a great interest for model-building and GUT-phenomenology.

4 \hspace{1em} The pseudoscalar decay constants and \( B_B \)

The decay constants \( f_P \) of a pseudoscalar meson \( P \) are defined as:

\[ (m_q + M_Q)(0|\bar{q}(i\gamma_5)Q|P) \equiv \sqrt{2}M_P^2 f_P, \]  

(18)

where in this normalisation \( f_\pi = 93.3 \text{ MeV} \). A rigorous upper bound on these couplings can be derived from the second-lowest superconvergent moment:

\[ \mathcal{M}^{(2)} \equiv \left. \frac{1}{2!} \left( \frac{\partial^2 \Psi_5(q^2)}{(\partial q^2)^2} \right) \right|_{q^2=0}, \]  

(19)

where \( \Psi_5 \) is the two-point correlator associated to the pseudoscalar current. Using the positivity of the higher-state contributions to the spectral function one can deduce [15]:

\[ f_P \leq \frac{M_P}{4\pi} \left\{ 1 + \frac{3 m_q}{M_Q} + 0.75 l \alpha_s + ... \right\}, \]  

(20)

where one should not misinterpret the mass-dependence in this expression compared to the one expected from heavy quark symmetry. Applying this result to the \( D \)-meson one obtains:

\[ f_D \leq 2.14 f_\pi. \]  

(21)

Although presumably quite weak this bound when combined with the recent determination to two loops [16]:

\[ \frac{f_{D^*}}{f_D} \simeq (1.15 \pm 0.04) f_\pi, \]  

(22)
implies
\[ f_{D*} \leq (2.46 \pm 0.09)f_\pi, \] (23)

which is useful for a comparison with the recent measurement of \( f_{D*} \) from WA75: \( f_{D*} \simeq (1.76 \pm 0.52)f_\pi \) and from CLEO: \( f_{D*} \simeq (2.61 \pm 0.49)f_\pi \). One cannot push however the uses of the moments to higher \( n \)-values in this \( D \)-channel in order to minimize the continuum contribution to the sum rule with the aim to derive an estimate of the decay constant because the QCD-series will not converge at higher \( n \)-values. The estimate of \[ [17] \] based on the lowest moment indeed suffers from the continuum sensitivity as a little change in the continuum threshold makes a big change of the estimate in such a way that the result becomes unreliable. In the \( D \)-channel the most appropriate sum rule is the Laplace sum rule. The results from different groups are consistent for a given value of the \( c \)-quark mass and lead to the average \[ [1814] \]:
\[ f_D \simeq (1.31 \pm 0.12)f_\pi \quad \Rightarrow \quad f_{D*} \simeq (1.51 \pm 0.15)f_\pi. \] (24)

For the \( B \)-meson one can either work with the Laplace moments or their non-relativistic variants. Given the previous value of \( M_b \) these different methods give consistent values of \( f_B \) though the one from the non-relativistic sum rule is very inaccurate due to the huge effect of the radiative corrections in this method. The average value of \( f_B \) is \[ [19] \]:
\[ f_B \simeq (1.60 \pm 0.26)f_\pi, \] (25)

while \[ [16] \]:
\[ \frac{f_{B*}}{f_B} \simeq 1.16 \pm 0.04, \] (26)

where the most accurate estimate comes from the “relativistic” Laplace sum rule. One could notice since the first result \( f_B \simeq f_D \) of \[ [18] \] a large violation of the scaling law expected from heavy-quark symmetry. Indeed this is due to the large \( 1/M_b \)-correction found from the HQET sum rule \[ [2021] \] and from the one in full QCD \[ [1914] \]:
\[ f_B \sqrt{M_b} \simeq (0.42 \pm 0.07) \text{ GeV}^{3/2} \left\{ 1 - \frac{(0.88 \pm 0.18) \text{ GeV}}{M_b} \right\}, \] (27)

which is due to the meson-mass gap \( \delta M \equiv M_B - M_b \) \[ [21] \] and to the continuum energy \( E_c \) \[ [1922] \] \( (E_c \simeq \frac{3}{4}\delta M \) \[ [14] \]):
\[ f_B \sqrt{M_b} \simeq \frac{1}{\pi} E_c^{3/2} \left\{ 1 - \frac{\delta M}{M_b} - \frac{3}{2} \frac{E_c}{M_b} + ... \right\}. \] (28)

One can notice that the apparent disagreement among different existing QSSR numerical results in the literature is mainly due to the different values of the quark masses used because the decay constants are very sensitive to that quantity as shown explicitly in \[ [16] \]. Indeed one can also exploit this sensitivity in order to deduce the value of the quark mass when the experimental measurement of these decay constants will become available.

Finally let me also mention that we have also tested the validity of the vacuum saturation \( B_B = 1 \) of the bag parameter using a sum rule analysis of the four-quark two-point correlator to two loops \[ [23] \]. We found that the radiative corrections are quite small.
Under some physically reasonable assumptions for the spectral function we found that the vacuum saturation estimate is only violated by about 15% giving:

\[ B_B \simeq 1 \pm 0.15. \]  
(29)

These previous results are in excellent agreement with the present lattice calculations [24].

## 5 Rare and semileptonic decays

One can extend the analysis done for the two-point correlator to the more complicated case of three-point function in order to study the form factors related to the \( B \to K^*\gamma \) and \( B \to \rho/\pi \) semileptonic decays. In so doing one can consider the generic three-point function:

\[ V(p, p', q^2) \equiv -i \int d^4x \int d^4y \, e^{i(p' \cdot y - py)} \langle 0 | T J_L(x) O(0) J_B^\dagger(y) | 0 \rangle \]  
(30)

where \( J_L, J_B \) are the currents of the light and \( B \)-mesons; \( O \) is the weak operator specific for each process (penguin for the \( K^*\gamma \) weak current for the semileptonic); \( q \equiv p - p' \).

The vertex obeys the double dispersion relation:

\[ V(p^2, p'^2, q^2) \simeq \int_{M_B^2}^{\infty} \frac{ds}{s - p^2 - i\epsilon} \int_{M_B^2}^{\infty} \frac{ds'}{s' - p'^2 - i\epsilon} \frac{1}{\pi^2} \operatorname{Im} V(s, s', q^2) + \ldots \]  
(31)

As usual the QCD part enters in the LHS of the sum rule while the experimental observables can be introduced through the spectral function after the introduction of the intermediate states. The improvement of the dispersion relation can be done in the way discussed previously for the two-point function. In the case of the heavy-to-light transition the only possible improvement whith a good \( M_q \)-behaviour at large \( M_q \) is the so-called hybrid sum rule (HSR) corresponding to the uses of the moments for the heavy-quark channel and to the Laplace for the light one [14\( \Gamma \)25]:

\[ \mathcal{H}(n, \tau') = \frac{1}{\pi^2} \int_{M_B^2}^{\infty} \frac{ds}{s^{n+1}} \int_0^\infty ds' \, e^{-\tau's'} \operatorname{Im} V(s, s', q^2). \]  
(32)

We have studied analytically the different form factors entering in the previous processes [26]. They are defined as:

\[
\langle \rho(p') | \bar{u} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = \left( M_B + M_p \right) A_1 e_\mu^* - \frac{A_2}{M_B + M_p} e^* p'(p + p')_\mu \\
+ \frac{2 V}{M_B + M_p} \epsilon_{\mu\nu\rho\sigma} p^\nu j^\rho j^\sigma,
\]

\[
\langle \pi(p') | \bar{u} \gamma_\mu b | B(p) \rangle = f_+(p + p')_\mu + f_-(p - p')_\mu,
\]

\[
\langle \rho(p') | \bar{s} \sigma_{\mu\nu} \left( \frac{1 + \gamma_5}{2} \right) q^\nu b | B(p) \rangle = i \epsilon_{\mu\nu\rho\sigma} e^* p^\rho j^\sigma F_1^{B - p},
\]

\[
+ \left\{ e_\nu^* \left( M_B^2 - M_p^2 \right) - e^* q(p + p')_\mu \right\} \frac{F_1^{B - p}}{2}.
\]  
(33)
We found that they are dominated by the effect of the light-quark condensate which dictates the $M_b$-behaviour of the form factors to be typically of the form:

$$F(0) \sim \frac{\langle \bar{d}d \rangle}{f_B} \left( 1 + \frac{\mathcal{I}_F}{M_b^2} \right),$$

(34)

where $\mathcal{I}_F$ is the integral from the perturbative triangle graph which is constant as $t_c^2 E_c / \langle \bar{d}d \rangle$ ($t_c$ and $E_c$ are the continuum thresholds of the light and $b$ quarks) for large value of $M_b$. It indicates that at $q^2 = 0$ all form factors behave like $\sqrt{M_b} \Gamma$ although the coefficient of the $1/M_b^2$ term is large. The study of the $q^2$-behaviours of the form factors indicates that with the exception of the $A_1$-form factor the $q^2$-dependence of the others is only due to the non-leading (in $1/M_b$) perturbative graph such that for $M_b \to \infty \Gamma$ these form factors remain constant from $q^2 = 0$ to $q^2_{\text{max}}$. The resulting $M_b$-behaviour at $q^2_{\text{max}}$ is the one expected from the heavy quark symmetry. The numerical effect of this $q^2$-dependence at finite values of $M_b$ is a polynomial in $q^2$ which mimics quite well the usual pole parametrization for a pole mass of about 6-7 GeV. The situation for the $A_1$ is drastically different from the other ones as here the Wilson coefficient of the $\langle \bar{d}d \rangle$ condensate contains a $q^2$-dependence and reads:

$$A_1(q^2) \sim \frac{\langle \bar{d}d \rangle}{f_B} \left( 1 - \frac{q^2}{M_b^2} \right),$$

(35)

which for $q^2_{\text{max}} \equiv (M_B - M_\rho)^2$ gives the expected behaviour:

$$A_1(q^2_{\text{max}}) \sim \frac{1}{\sqrt{M_b}}.$$  

(36)

One can notice that the $q^2$-dependence of $A_1$ is in complete contradiction with the pole behaviour as has been also noticed in the numerical analysis of [27]. It is urgent and important to test this feature experimentally. One can finally notice that due to the overall $1/f_B$ factor all form factors have a large $1/M_b$-correction.

In the numerical analysis we obtain at $q^2 = 0$ the value of the $B \to K^*\gamma$ form factor:

$$F_1^{B\to \rho} \simeq 0.27 \pm 0.03, \quad \frac{F_1^{B\to K^*}}{F_1^{B\to \rho}} \simeq 1.14 \pm 0.02,$$

(37)

which leads to the branching ratio $(4.5 \pm 1.1) \times 10^{-5}$ in perfect agreement with the CLEO data and with the estimate in [28]. One should also notice that in this case the coefficient of the $1/M_b^2$ correction is very large which makes the extrapolation of the $c$-quark results to higher values of the quark mass dangerous. This extrapolation is often done in some lattice calculations.

For the semileptonic decays QSSR give a good determination of the ratios of the form factors with the values:

$$\frac{A_2(0)}{A_1(0)} \sim \frac{V(0)}{A_1(0)} \simeq 1.18 \pm 0.04$$

$$\frac{A_1(0)}{F_1^{B\to \rho}(0)} \simeq 1.19 \pm 0.05$$

$$\frac{A_1(0)}{f_+(0)} \simeq 1.40 \pm 0.06.$$  

(38)
Combining these results with the “world average” value of $f_+(0) = 0.23 \pm 0.02$ or the one of $F_{1B^{-}}(0)$ one can deduce the rate and polarization:

$$\Gamma_\pi \simeq (3.0 \pm 0.5)|V_{ub}|^2 \times 10^{12} \text{s}^{-1}$$
$$\Gamma_\rho / \Gamma_\pi \simeq 1.3 \pm 0.2$$
$$\alpha \equiv \frac{2\Gamma_L}{\Gamma_T} - 1 \simeq -(0.87 \pm 0.10). \quad (39)$$

These results are much more precise than the ones from a direct estimate of the absolute values of the form factors due to the cancellation of systematic errors in the ratios. They indicate that we are on the way to reach $V_{ub}$ with a good accuracy. Also here the ratio between the widths into $\rho$ and into $\pi$ is about 1.5 while in different pole models it ranges from 3 to 10. For the asymmetry one obtains a large negative value of $\alpha$ contrary to the case of the pole models.

### 6 Slope of the Isgur–Wise function and $V_{cb}$

Let me now discuss the slope of the Isgur–Wise function. Taron–de Rafael [29] has exploited the analyticity of the elastic $b$-number form factor $F$ defined as:

$$\langle B(p') | \bar{b} \gamma^\mu b | B(b) \rangle = (p + p')^\mu F(q^2) \quad (40)$$

which is normalized as $F(0) = 1$ in the large mass limit $M_B \simeq M_D$. Using the positivity of the vector spectral function and a mapping in order to get a bound on the slope of $F$ outside the physical cut they obtained a rigorous but weak bound:

$$F'(vv' = 1) \geq -6. \quad (41)$$

Including the effects of the $\Upsilon$ states below $\bar{B}B$ thresholds by assuming that the $\Upsilon \bar{B}B$ couplings are of the order of 1 the bound becomes stronger:

$$F'(vv' = 1) \geq -1.5. \quad (42)$$

Using QSSR we can estimate the part of these couplings entering in the elastic form factor. We obtain the value of their sum [30]:

$$\sum g_{\Upsilon \bar{B}B} \simeq 0.34 \pm 0.02. \quad (43)$$

In order to be conservative we have considered the previous estimate within a factor 3 larger. We thus obtained the improved bound

$$F'(vv' = 1) \geq -1.34, \quad (44)$$

but the gain is not much compared with the previous one. Using the relation of the form factor with the slope of the Isgur–Wise function which differs by $-16/75\log \alpha_s(M_t)$ [31] one can deduce the final bound:

$$\zeta'(1) \geq -1.04. \quad (45)$$
However, one can also use the QSSR expression of the Isgur–Wise function from vertex sum rules [21] in order to extract the slope analytically. The physical IW function reads:

\[
\zeta_{\text{phys}}(y \equiv v'v) = \left(\frac{2}{1 + y}\right)^2 \left\{ 1 + \frac{\alpha_s}{\pi} f(y) - \langle \bar{d}d \rangle \tau^3 g(y) + \langle \alpha_s G^2 \rangle \tau^4 h(y) + g \langle \bar{d}Gd \rangle \tau^5 k(y) + \ldots \right\},
\]

where \( \tau \) is the Laplace sum rule variable and \( f, h \) and \( k \) are analytic functions of \( y \). From this expression one can derive the analytic form of the slope [30]:

\[
\zeta'_{\text{phys}}(y = 1) \simeq -1 + \delta_{\text{pert}} + \delta_{\text{NP}},
\]

where at the \( \tau \)-stability region: \( \delta_{\text{pert}} \simeq -\delta_{\text{NP}} \simeq -0.04 \), which shows the near-cancellation of the non-leading corrections. Adding a generous 50% error of 0.02 for the correction terms we finally deduce:

\[
\zeta'_{\text{phys}}(y = 1) \simeq -1 \pm 0.02.
\]

Using this result in different existing model-parametrizations we deduce the value of the mixing angle:

\[
V_{cb} \simeq \left(\frac{1.48 \text{ps}}{\tau_b}\right)^{1/2} \times (37.3 \pm 1.2 \pm 1.4) \times 10^{-3},
\]

where the first error comes from the data and the second one from the model dependence.

Let us now discuss the effects due to the \( 1/M \) correction. It has been argued recently that this effect can lower the Isgur–Wise function to a value \( 0.89 \pm 0.03 \) at \( y = 1 \) [32] such that the extracted value of \( V_{cb} \) using an extrapolation until this particular point will also increase by 11%. However, the data from different groups near this point are very inaccurate and lead to an inaccurate though model-independent result. Moreover, in order to see the real effect of the \( 1/M \) correction one can combine this previous result at \( y = 1 \) with the sum rule estimate of the relevant form factor at \( q^2 = 0 \) which is about \( 0.53 \pm 0.09 \) [25]. With these two extremal boundary conditions and using the linear parametrization:

\[
\zeta = \zeta_0 + \zeta'(y - 1),
\]

we can deduce the slope:

\[
\zeta' \simeq -(0.72 \pm 0.2).
\]

It indicates that the \( 1/M \) correction tends also to decrease \( \zeta' \) which implies that \( \Gamma \) for larger values of \( y \) where the data are more accurate the increases of \( V_{cb} \) is weaker \((+3.7\%)\) than the one at \( y = 1 \) which leads to the final estimate:

\[
V_{cb} \simeq \left(\frac{1.48 \text{ps}}{\tau_b}\right)^{1/2} \times (38.8 \pm 1.2 \pm 1.5 \pm 1.5) \times 10^{-3},
\]

where the new last error has been induced by the error from the slope. This result is more precise than the one obtained at \( y = 1 \) while the model-dependence only brings a relatively small error. It also shows that the value from the exclusive channels is lower than that from the inclusive one which is largely affected by the large uncertainty in the mass definition which enters in its fifth power. Previous results for the slope and for \( V_{cb} \) are in good agreement with the new CLEO data presented in this meeting.
7 Hybrid and $B_c$-mesons

Let me conclude this talk by shortly discussing the masses of the hybrid $QGQ$ and the mass and decays of the $B_c$-mesons. Hybrid mesons are interesting due to their exotic quantum numbers. Moreover, it is not clear if these states are true resonances or instead they only manifest as a wide continuum. The lowest $\varepsilon Gc$ states appear to be a $1^{--}$ of a mass around 4.1 GeV [3]. The available sum-rule analysis of the $1^{--}$ state is not very conclusive due to the absence of stability for this channel. However, the analysis indicates that the spin one states are in the range 4.1–4.7 GeV. Their characteristic decays should occur via the $\eta'$ $U(1)$-like particle produced together with a $\psi$ or $\eta_c$. However, the phase-space suppression can be quite important for these reactions. The sum rule predicts that the $0^{--}$, $0^{++}$ $\varepsilon Gc$ states are in the range 5–5.7 GeV, i.e. about 1 GeV above the spin 1.

We have estimated the $B_c$-meson mass and coupling by combining the results from potential models and QSSR [8]. Potential models predict:

$$M_{B_{c}} = (6255 \pm 20) \text{ MeV}, \quad M_{B_{c}^*} = (6330 \pm 20) \text{ MeV} \quad (53)$$

$$M_{\Lambda[bcu]} = (6.93 \pm 0.05) \text{ GeV}, \quad M_{\Omega[bcu]} = (7.00 \pm 0.05) \text{ GeV}$$

$$M_{\Xi^1(cbc)} = (3.63 \pm 0.05) \text{ GeV}, \quad M_{\Xi^1(bb)} = (10.21 \pm 0.05) \text{ GeV}$$

which are consistent with but more precise than the sum-rule results. The decay constant of the $B_c$ meson is better determined from QSSR. The average of the sum rules with the potential model results reads:

$$f_{B_{c}} \simeq (2.94 \pm 0.12) f_{\tau}, \quad (54)$$

which leads to the leptonic decay rate into $\tau \nu_{\tau}$ of about $(3.0 \pm 0.4) \times (V_{cb}/0.037)^2 \times 10^{10} \text{ s}^{-1}$.

We have also studied the semileptonic decay of the $B_c$ mesons and the $q^2$-dependence of the form factors. We found that in all cases the QCD predictions increase faster than the usual pole dominance ones. The behaviour can be fitted with an effective pole mass of about 4.1–4.62 GeV instead of the 6.33 GeV one expected from a pole model. Basically, we also found that the each exclusive channel has almost the same rate of about 1/3 of the leptonic one. Detections of these particles in the next $B$-factory machine will serve as a stringent test of the results from the potential models and sum rules analysis.

8 Conclusion

We have shortly presented different results from QCD spectral sum rules in the heavy-quark sector which are useful for further theoretical studies and complement the results from lattice calculations or and heavy-quark symmetry. For the experimental point of view, QSSR predictions agree with available data but they also lead to some new features which need to be tested in forthcoming experiments.
References


