Performance simulation of the cathode strip chambers for CMS End Cap Muon System

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Abstract

A model has been considered of the Cathode Strip Chamber (CSC) exposed to the muon stream of high intensity (70 Hz/cm²) originating in the crossing point of beams of the LHC collider. To reproduce the experimental conditions, the intensity of the muon stream in our model had the impulse structure of 40 MHz rate. Complete simulation of registration process from initial ionization to the final signals was performed for muons that cross the 6-layers trapezoidal CSC. The anode and cathode current signals were used for evaluation of the hits and for simulation of the cathode pulse shape in the Switch Capacity Array (SCA). The Anode and Cathode Local Charged Tracks as well as the 1-level trigger solutions were generated for reconstruction of the tracks and for approximate evaluation of their space and time coordinates by means of the logic approved for CMS End Cap Muon System. More exactly, the time and spatial coordinates of detected muons have been determined from fitting of the SCA information.
1 Introduction

In our previous report [1], the results of performance simulation of the prototype P1 of Cathode Strip Chamber (CSC) in muon beam of low intensity have been presented. The influence of several factors such as the electron attachment, the noise of electronics on time and spatial resolutions of the detector prototype have been studied. Our results were based on data obtained in the experiment [2,3]. Its authors had tested two CSC prototypes in a muon beam at CERN. These prototypes were designed to represent two small sections of the ultimately large CSCs that will be built for the End Cap Muon System of the CMS Detector. Later, the full scale prototype P1 was constructed and tested at Fermilab with cosmic rays. Both the experimental results and our simulation led to the conclusion that the chamber will meet all the requirements concerning time and spatial resolutions. But the experimental research on P1 has been hindered by the small intensity of muons ($\approx 10^{-2}$ Hz/cm$^2$). However, the rates of background are expected to be as high as 70 Hz/cm$^2$ [2]. Therefore it is necessary to estimate the influence of the background on chamber performance. This paper deals with the problems one faces using the chamber in such severe and more realistic background conditions.

2 The procedure of calculation.

The chambers chosen for inner and upper rings of the CMS End Cap Muon System are 6 - layer trapezoidal cathode strip chambers. To be more definite, we have considered here 2 typical types of them (the chambers ME3/1 and ME3/2).

The Figure 1 presents the schematic view of our model experimental setup.

The muons (P = 150 GeV/c) created in the LHC p - p crossing point O transverse the working volume of the chamber. The muons are uniformly distributed in the plane $X - Y$ with density $P_b = 70$ Hz/cm$^2$. The distance H from the point O to the $X - Y$ plane was set to be equal to 900cm.

![Figure 1. A schematic view of the experimental setup.](image-url)
The parameters of chambers are presented in the Table 1.

The even strip layers of each chamber are rotated by the angle equal to half strip pitch \((0.5 \Delta \phi)\) in the \(X - Y\) plane to improve the spatial resolution.

The shift of wires between different layers was assumed to be zero.

The distribution of initial ionization along the track was calculated by the program HEED [4]. While drifting to an anode wire, the ionization electrons interact with molecules of the gas, deviate from their paths along the electric field lines and can even be captured. Approaching the anode wires, electrons create avalanches. The principal characteristics of the gas mixture, the electron drift velocity and diffusion coefficient, that determine the transportation of electrons to the anode wires, were calculated by the program BOLNEW [5] for the gas mixture of 40\% Ar, 10\% CF\(_4\), 50\% CO\(_2\).

To keep the gas gain constant, the fraction of initial ionization \(p\), captured in the gas mixture while drifting, and the average number of electrons \(a\) created by each survived initial electron in avalanches near anode wires, were chosen according to the relation

\[
a(1 - p) = 80000.
\]

This resulted in constant measured average signal amplitude for arbitrary choice of \(p\) value. (See Table 1). The difference of amplitudes for ME3/1 and ME3/2 is due to the difference in average entry angles of muons in these two chambers.

To reduce the time needed for calculation, the following simplified procedure has been used. The random selection of initial electrons has been performed before the beginning of the drifting simulation. Each survived electron then drifted to an anode wire and created an avalanche (a cluster) on its surface. The charge created by one electron was determined according to an exponential distribution. The Figure 2 illustrates the trigger efficiency \(E\) as a function of the electron capture rate \(p\). Its shape leads to the conclusion that the chamber performance degrades essentially only at large values of \(p\).

![Figure 2. The trigger efficiency \(E\) as a function of the electron capture rate \(p\).](image-url)
All calculations here were performed for \( p = 60\% \). Such value of \( p \) is not very optimistic but yet acceptable. We have obtained \( E = 97.2\% \) for ME3/1 and 95.3\% for ME3/2. The noise of electronics here was assumed to be equal 0.

We have neglected totally the diffusion of ions in clusters. The drift velocity of a positive cluster

\[ v^{(+)} = \mu E. \]

Here

\( E \) is the electric field tension,
\( \mu = 1.3 \text{ cm}^2/(\text{V} \cdot \text{s}) \) - the mobility of ions.

The temporal behavior of the anode and cathode signals produced by each cluster is totally determined by their following parameters: the charge \( Q \), the origin time \( t_0 \), the coordinate along the wire \( X \), the angle \( \theta \) in the plane \((Y - Z)\) and the number of wire \( w \), on the surface of which the avalanche was created.

The two functions

\[ F_a(t, g; t_0, w, X, \theta) \]

and

\[ F_c(t, s; t_0, w, X, \theta), \]

normalized to the charge value \( Q = 1 \text{ fC} \), determine the temporal and spatial behavior of one-cluster signals, and they are of special interest for this research.

These functions have been used for two purposes: for simulation of the signals produced by the ionization tracks, and for track reconstruction from the Switch Capacity Array (SCA) information.

\( F_a \) and especially \( F_c \) depend on the cluster position in a chamber. To take this into account in calculation of \( F_a \) and \( F_c \), the chamber has been divided into 5 small sections in \( Y \) - direction, and for \( F_a \) also into 7 sections in \( X \) - direction.

We have totally neglected the dependence of \( F_c \) on strip rotation around the \( Y \) - direction (\( \pm 10^\circ \) for ME3/1 and \( \pm 5^\circ \) for ME3/2).

The functions \( F_c \) and \( F_a \) are in fact the results of convolution of the following 3 functions.

1. The current \( I(t, w) \) or \( I(t, s) \) in the wire \( w \) or strip \( s \) created by a cluster drifting to the cathode in a chamber module.

These currents are related to the induced charges \( Q_{\text{ind}}^a \) and \( Q_{\text{ind}}^c \) by the formulas:

\[ I(t, w) = \frac{d}{dt} Q_{\text{ind}}^a(w, X, Y, Z) \]

\[ I(t, s) = \frac{d}{dt} Q_{\text{ind}}^c(s, X, Y, Z) \]

where \( X, Y, Z \) are the coordinates of the cluster at the moment \( t \).

The value of induced charge \( Q_{\text{ind}}^a \) on anode wires was calculated by solving a two-dimensional problem in quadrupole approximation.
The charges $Q_{\text{ind}}$ for the cathode strips have been taken from a table, generated in advance by means of a special program. This program calculates the induced charges on the strips directed perpendicular to the anode wires for any given position $X, Y, Z$ of a point-like charge in a chamber module.

Figure 3. The electric circuit model for calculation of response of wires (strips).

2. The response of a strip (wire) and its neighbors to a $\delta$ - function shaped current in a point of a strip (wire) situated above the avalanche point.

Calculating these functions one must consider the chamber as a system with distributed electric parameters. The procedure of calculation was described in more detail in [1]. To model such a system, each strip (wire) has been divided into 35 (21) small segments (Figure 3). Each segment $k$ was represented by a capacitor $C_k$ and connected with adjacent capacitor along the strip (wire) by inductors $L_k$ and $L_{k+1}$. The capacitors $C_k^m$ ($m = 1, 2, \ldots 5$) were also included between each pair of strips (wires) $n$ and $n \pm m$. The method of calculation of these capacities can be found in [6]. For simplicity, only the capacities between adjacent strips (wires) $C_k^1$ are shown on the Figure 3. In case of strips, values of all parameters depend on position of the segment along the strip.

All calculations here were performed for the case when the wave resistance of a strip (wire) and the input resistance of the preamplifier $R$ were equal.

We have totally ignored the influence of the cathode signals on the anode signals and vice versa (i.e. the wire - to - strip capacities were assumed to be 0).

3. The response of the chamber electronics. The response function $R_a$ of the anode electronics was a fifth order pole with one zero:

$$R_a(t) = C_a \frac{\partial}{\partial t} \left( t^4 \exp(-t/\tau_a) \right),$$

$$\tau_a = 7.5 \text{ ns}.$$  

The anode signal peaking time was 30 ns, maximum value of $R_a = 1$.

The response function of the cathode electronics was the same as in the Buckeye chip [8]. Its design has 5 shaping poles with 1 zero for tail cancellation. In this simulation
the numerical response presentation was used. It was copied from enlarged histogram of Buckeye chip response (Fig.4.4.4a in [8]). The tail of this response was taken in linear form so that the total area of the signal was equal to zero. This resulted in long small negative tails of triangular form in the signals from \( t = 400 \) ns and up to 4000 ns. More realistic case of tail of exponential form affects negligibly the results of simulation but requires essentially more computer time for calculation.

The total amplitude of a signal on the wire \( w \) (strip \( s \)) at the moment \( t \) is the sum of signals from all the clusters drifting in the given layer of a chamber.

\[
F(t, w) = \sum_{i=1}^{n} Q_i F_a(t, w; t_0^i, w^i, X^i, \Theta^i)
\]

\[
F(t, s) = \sum_{i=1}^{n} Q_i F_e(t, s; t_0^i, w^i, X^i, \Theta^i)
\]

Here \( Q_i \) is the charge of a cluster \( i \).

There is no need to simulate the correlations between the signals in different layers of a chamber. We obtain them automatically.

To reduce the number of anode channels in real experiment, the wires were connected into groups.

Our model reproduces this procedure only approximately. Each wire was connected to its own preamplifier. The connection of channels was done actually after anode preamplifiers. In such case the output anode signal \( F(t, g) \) for any group \( g \) is simply the sum of signals from the wires in this wire group.

The following procedure was used to simulate the contribution of electronics noise into measured signal.

Let

\[
I_{\text{in}}(t) = \frac{q}{\sqrt{n\Delta \Delta}} \sum_{-\infty}^{t \leq t} \alpha_i \delta(t - t_i)
\]

be the random sequence of input current pulses.

Here

- \( t_i \) - are the origin times of the pulses of current, Poisson-distributed,
- \( n \) - is the average number of pulses in unit time interval \( \Delta \),
- \( \alpha_i \) are normally distributed with average value 0 and dispersion 1.
- \( q \) - some constant charge.

Such current on input produces the output (measured) charge
\[ Q(t) = \frac{q}{\sqrt{n \Delta}} \sum_{t_i}^{t_i < t} \alpha_i R(t - t_i), \]

\( R(t) \) - is the anode or cathode response function.

The charge \( Q(t) \) when squared and averaged over a large time interval \( T \) gives the following value of equivalent noise charge \( ENC \)

\[ ENC^2 = \frac{1}{T} \int_0^T Q^2(t) \, dt = \frac{q^2}{\Delta} \int_0^\infty R^2(t) \, dt. \]

This relation determines the value of \( q \).

One restriction was imposed on value of \( n \):

\[ n t_p \gg 1. \]

Here \( t_p \) is the peaking time of anode (cathode) response function.

The following typical experimental values of \( ENC \) were taken to calculate the noise of electronics, contributing to the real output of anode and cathode signals [7].

<table>
<thead>
<tr>
<th>ENC [fC]</th>
<th>ME3/1</th>
<th>ME3/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>anode</td>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>cathode</td>
<td>1.4</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The last remark. The study of influence of the edge effects on temporal and spatial resolution of the chambers is out of the scope of this research. Throughout all this paper we have distinguished the muons that cross the full aperture of a chamber from those that cross it in the registration window. The ratio of the numbers \( N_f \) and \( N_w \) of these two types of muons was 1.19.

3 On - line considerations.

A comprehensive description of anode and cathode electronics can be found in the Muon Project TDR[8] and in the Level-1 Trigger TDR[9]. Only details affecting simulation are mentioned here.

3.1 The hits.

To find the time and spatial positions of the anode hits, the continuous output signal \( F(t, g) \) of a preamp/shaper is to be processed for each wire group \( g \). The following discriminator logic has been simulated.
• The flag “busy” is switched off, and the current \( F(t, g) \) exceeds the pre-set threshold (5 fC) at the time moment \( t_0 \) for the first time. Then the flag “busy” is switched on and the electronics of this wire group is closed for registration of subsequent tracks during the time window \( t_w = 150 \text{ ns} \).

• The flag “busy” is switched on, and the signal \( F(t, g) \) crosses zero level at time moment \( t \) \((t - t_0 \leq t_w)\) for the first time. Then the time coordinate of the anode hit is taken to be equal to the value of the anode bunch counter clock nearest to \( t \). Its spatial position is \( g \). The flag “busy” is switched off and the channel is ready to process signals.

• No zero crossing has been observed, but the time window is exhausted. The flag “busy” for the group \( g \) is switched off.

To find the temporal and spatial position of the cathode hits, the comparator logic adopted for CMS End Cap Muon system was implemented.

To reproduce the time structure of the bunch, the continuous output signals of the preamp/shapers for all strips of a chamber have been sampled at 40 MHz rate. For every fixed sample (bunch), the charge profile \( F(t, s) \) has been analyzed as a function of the strip number \( s \). To find positions of the cathode hits, all the strips with peaks of the profile were searched. Only the strips with peaks that exceeded the pre-set threshold \( q = 5 \text{ fC} \) were selected. The half-strip localization of the hit position can be performed by comparison of the charges for neighboring strips (the strip \( s - 1 \) and the strip \( s + 1 \)). In case of \( F(t, s - 1) \leq F(t, s + 1) \) the hit coordinate \( h \) was taken to be \( 2s \), otherwise \( 2s - 1 \).

In the hit-searching procedure described above, one needs to implement two different bunch counting clocks. Both have the same frequency (40 MHz), but different start moments. The shifts in bunch positions of these clocks relative to the LHC bunch counting clock have been determined by minimizing the number of errors in ALCT and CLCT bunch identifications (see their definition below).

To model the performance of the Switch Capacity Array (SCA), the following procedure was adopted.

Let some CLCT be registered with the coordinates \((s, b)\):

\( s \) is the number of the strip where the hit was registered,

\( b \) is the number of the bunch to which it belongs.

A fragment \((9 \times 8)\) of voltage samples (of even bunches only) is extracted from the non-stopping sample stream and is saved for all layers of a chamber. This array covers such a range of the strips:

\[ s - 4, ..., s, ..., s + 4 \]

in the time interval (in bunches):

\[ b_0, b_0 + 2, ..., b_0 + 14, \]
3 ≤ b - b₀ ≤ 4.

A typical picture of a hit shape is presented in the Table 3a for one layer of a chamber. It will be used later for more precise determination of the track’s temporal and spatial coordinates.

3.2 ALCT and CLCT.

The other objects we need to extract from the stream of hits are so called anode (ALCT) and cathode (CLCT) local charged tracks. They are the chains of hits, created by a muon while crossing the CSC. Both time and spatial correlations in distribution of the hits are exploited in this procedure.

In general outline we follow here the algorithm adopted for the CMS End Cap Muon System [8].

The pattern of tracks (envelope) was projected onto each half - strip (wire group) in the base layer (the third layer in this research). The half - strips (wire groups) situated inside of the envelope (the black breaks on the Fig. 4) indicate the road allowed for positions of the hits included in CLCT (ALCT).

Our choices of the envelopes for the chambers ME3/1 and ME3/2 are presented on the Figure 4.

![Figure 4. The anode (A) and cathode (C) envelopes.](image)

The consequent time correlations for allowed hits of ALCT are demanded. There must be not less than 2 allowed hits that belong to the bunch \( b \) and not less than 4 allowed hits for the pair of bunches \( b \) and \( b + 1 \).

For CLCT not less than 4 allowed hits in bunch \( b \) are demanded. To reduce repetitions in selection of the same CLCT in later bunches, the dead time was set \( t_s = 200 \text{ ns} \).

When these conditions are fulfilled, one has an ALCT (CLCT) that belongs to the bunch \( b \). Its spatial coordinate is then taken to be equal to the number of the wire group (half - strip) in the base layer.

The number of different layers \( n_L \) occupied by allowed hits was assigned in our simulation as the first priority number of ALCT (CLCT). The larger \( n_L \), the higher is the track priority. The total number of allowed hits \( n_T \) was used in simulation as the second priority number of ALCT (CLCT). In case when \( n_L \) are equal for two LCT the second priority numbers are compared. The
larger \( n_t \), the higher is the track priority. If LCTs create a cluster (the group of adjacent LCTs), only one LCT with highest priority is selected in each cluster. We have now a set of isolated ALCTs (CLCTs). Only not more than two ALCT (CLCT) with highest priorities are selected from these sets for further consideration in each bunch.

Two streams of local charge tracks (the ALCT and CLCT - streams) are supplemented in simulation by the third stream of muon’s generated tracks (the GT - stream). Let it be

- \( b \) - the number of the bunch to which the GT belongs,
- \( b_a \) and \( b_c \) - the same for ALCT and CLCT.
- \( g \) and \( g_a \) - the \( Y \) coordinates of the GT and ALCT (the wire group),
- \( h \) and \( h_c \) - the \( X \) coordinates of the GT and CLCT (in half - strips).

Our aim now is to extract from ALCT and CLCT streams the candidates with coordinates that suit best to the GT - coordinates. What (ALCT,CLCT) - pair is the best representative of a GT is not always a clear question. The following procedure was implemented here. The search of candidates is performed in the time window \( w_a = b \pm 2 \) for ALCT and in \( w_c = b \pm 3 \) - for CLCT. In cases when in both windows there were LCTs, the following algorithm was implemented for the search of candidates. Each subsequent step here was performed only if all previous ones were not successful.

1. **Good GT reconstruction (no mistakes).** Such a pair (ALCT,CLCT) is searched for that satisfies the conditions:
   
   \[
   \begin{align*}
   b_a &= b \\
   |b_c - b| &\leq 1 \\
   |g_a - g| &\leq 1 \\
   |h_c - h| &\leq 1 
   \end{align*}
   \]

2. **Mistakes in anode bunch determination.** Such a pair (ALCT,CLCT) is searched for that satisfies the conditions:
   
   \[
   \begin{align*}
   |b_a - b| &= 1 \\
   |b_c - b| &\leq 1 \\
   |g_a - g| &\leq 1 \\
   |h_c - h| &\leq 1 
   \end{align*}
   \]

3. **Mistakes in cathode bunch determination.** Such a pair (ALCT,CLCT) is searched for that satisfies the conditions:
   
   \[
   \begin{align*}
   |b_a - b| &\leq 1 \\
   |b_c - b| &= 2 \\
   |g_a - g| &\leq 1 \\
   |h_c - h| &\leq 1 
   \end{align*}
   \]
Mistakes in anode coordinate determination. Such a pair (ALCT, CLCT) is searched for that satisfies the conditions:

\[ b_a = b \]
\[ |b_c - b| \leq 1 \]
\[ |g_a - g| > 1 \]
\[ |h_c - h| \leq 1 \]

Mistakes in cathode coordinate determination. Such a pair (ALCT, CLCT) is searched for that satisfies the conditions:

\[ b_a = b \]
\[ |b_c - b| \leq 1 \]
\[ |g_a - g| \leq 1 \]
\[ |h_c - h| > 1 \]

All other mistakes. The best representative of the GT is the pair (ALCT, CLCT) that includes ALCT with the lowest value of \(|b_a - b|\). In case if there are two or more such ALCTs, we choose among them the one with lowest value of \(|g_a - g|\). The same procedure is performed with CLCTs. We choose CLCT that gives the lowest value of \(|b_c - b|\). If there are two or more such CLCTs we choose among them the one with lowest value of \(|h_c - h|\).

In case that in the anode (cathode) window there are no ALCTs (CLCTs), the procedure described above is performed only for the CLCT (ALCT) - stream.

One can compare now the temporal and spatial coordinates of a GT and its best on - line (ALCT, CLCT) representative.

The results of our calculations for ME3/1 are presented in a map (Table 2).

The Table B can be drawn from this map and from the similar one for ME3/2:

<table>
<thead>
<tr>
<th></th>
<th>ME3/1</th>
<th>ME3/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N (%)</td>
<td>N (%)</td>
</tr>
<tr>
<td>no mistakes</td>
<td>28812</td>
<td>18091</td>
</tr>
<tr>
<td>anode mistakes:</td>
<td>158</td>
<td>67</td>
</tr>
<tr>
<td>( b_a = b + 1 )</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>anode mistakes:</td>
<td>57</td>
<td>48</td>
</tr>
<tr>
<td>( b_a = b - 1 )</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>cathode mistakes:</td>
<td>169</td>
<td>332</td>
</tr>
<tr>
<td>( b_c = b + 2 )</td>
<td>0.6</td>
<td>1.7</td>
</tr>
<tr>
<td>cathode mistakes:</td>
<td>33</td>
<td>40</td>
</tr>
<tr>
<td>( b_c = b - 2 )</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>anode coordinate mistakes</td>
<td>14</td>
<td>27</td>
</tr>
<tr>
<td>cathode coordinate mistakes</td>
<td>59</td>
<td>109</td>
</tr>
<tr>
<td>no ALCT for track</td>
<td>131</td>
<td>22</td>
</tr>
<tr>
<td>no CLCT for track</td>
<td>101</td>
<td>51</td>
</tr>
<tr>
<td>other mistakes</td>
<td>201</td>
<td>200</td>
</tr>
<tr>
<td>all tracks (N_w)</td>
<td>29735</td>
<td>18987</td>
</tr>
</tbody>
</table>

11
The part of tracks reconstructed without mistakes (the efficiency) depends strongly on beam intensity (strip or wire group occupancy). The Figure 2 illustrates this dependence. Because the area of a strip (wire group) in ME3/2 is $2 - 3$ times larger than that of ME3/1, the efficiency of ME3/2 will be lower.

The results presented in Table B don’t differ essentially from those obtained in case of noiseless electronics. For efficiency we had the following values: $E = 97.2\%$ for ME3/1 and 95.3\% for ME3/2.

### 3.3 L1 - trigger solutions.

In a real experiment we have only two streams of local charged tracks. Both streams (especially the CLCT) are rather superfluous. Due to the long tail of the cathode signal of a track, there are a lot of repetitions in registration of CLCTs even for large values of the dead time (200 ns). On this stage the volume of disk storage required for recording of the information exceeds 10 KB/muon. It is almost 6 times more than 1.76 KB needed for saving of the GT, ALCT, CLCT and SCA - contents related to one muon. One needs to reduce the abundance of this information.

We follow here the way chosen for the CMS Muon End Cap System.

Suppose that $|b_h - b_c| \leq 1$ for some pair (ALCT,CLCT). We will call such objects registered tracks (RTs) that belong to the bunch $b_a$. Not more than 2 ALCTs for the bunch $b$ and not more than 6 CLCTs can create RTs in one bunch. At the same time there can be any number of GTs in this bunch.

A positive trigger solution L1 is generated only for a bunch with one or more RTs.

If it is the case for some bunch, then all the information related to the RTs belonging to this bunch ($b_a, b_c, g_a, h_c$, the priority numbers and the SCA - information for CLCTs) is saved for further off-line consideration.

<table>
<thead>
<tr>
<th>GT</th>
<th>ALCT</th>
<th>CLCT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

The frequencies of different cases that we had observed for the bunches with RTs are presented in the Table C for ME3/1.

The following two parameters characterize the quality of the trigger system performance.
1. Trigger efficiency. It is the fraction of GTs that has been registered \((b_a = b)\).

This number can be extracted from the Table B mentioned in the previous part. It is equal to 96.9\% for ME3/1 and 95.3\% for ME3/2. The number of lost GTs will be then 3.1\% (4.7\%) for ME3/1 (ME3/2).

2. The frequency of bunches \(\gamma\) with extra RTs.

This number can be evaluated in a special run when we have \(N_w = N_f\).

Let

\(N_r\) be the number of bunches with extra RTs (no GTs),

\(N_g\) - the total number of bunches with RTs.

Then \(\gamma\) will be

\[
\gamma = \frac{N_r}{N_g}
\]

In case of ME3/1 (Table C) our simulation gives \(\gamma = 3.6\%\).

For ME3/2 it was only 2.8\%.

The parameter \(\gamma\) was calculated here for the case ENC = 0.

The main source of extra RTs (GT = 0, ALCT = CLCT = 1, Table C) is the occasional time coincidence of extra ALCT and CLCT from a real track. An extra ALCT here is the one with mis-determined bunch position of a track or a repeated registration of the same ALCT in a later bunch. Let only one RT be in a given bunch. Our calculation of the probability for a pair (ALCT, CLCT) to produce an extra track for different values of anode and cathode priority numbers \(n^{(a)}_l\) and \(n^{(c)}_l\) is presented in the Table D. It is rather small for \(n^{(a)}_l = 6\), but we can not distinguish between real and extra RTs completely. The most reliable method for reduction of extra RTs is increasing of dead time for CLCTs. The second one - the increasing of the cathode hits registration threshold.

<table>
<thead>
<tr>
<th>(n^{(a)}_l)</th>
<th>(n^{(c)}_l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.76</td>
</tr>
<tr>
<td>5</td>
<td>0.39</td>
</tr>
<tr>
<td>6</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Let

\(n_a\) - be the number of ALCTs in the bunch \(b\),

\(n_c\) - the number of CLCTs in the bunches \(b - 1, b, b + 1\).

In case when

\[n_an_c > 1\]

13
an ambiguity arises. How shall we combine the ALCTs and CLCTs in the RTs to determine properly the numbers and the positions of all GTs?

The Table C shows that there are 2 main sources of ambiguities.

1. \( n_a = 1, n_c = 2 \). The part of bunches with such RTs was 9.4% for ME3/1. The source of such cases is an occasional time coincidence of a real track and a cathode signal tail from an earlier track. Usually one can recognize the true track coordinates if the SCA contents for both CLCTs are analyzed. The number of such RTs, as well as the number of extra RTs (GT = 0, ALCT = CLCT = 1, Table C), can be reduced essentially if one increases the dead time for CLCTs up to 250 to 300 ns. The second possibility for reduction of CLCT repetitions is the increasing of the threshold for cathode hits registration (current value - 5 fC).

2. \( n_a = 2, n_c = 2 \). (1.1% bunches with RTs). The source of such cases is an occasional time coincidence of two real tracks. We can hardly hope to distinguish between two possibilities that we have for two tracks if we use ALCTs and CLCTs - information only. In further consideration, to avoid this problem we will use our knowledge of real coordinates of tracks.

4 Off-line consideration.

4.1 The Switch Capacity Array. General details.

We are now ready for more correct evaluation of the cathode hit coordinates by fitting the SCA shape.

Each bump in the SCA shape will be approximated by the signal from two clusters originated in the time moment \( t_0 \) on the surface of a wire \( w \) with coordinate \( X \) along it and moving in opposite directions (to upper and lower cathodes):

\[
F(t, s) = Q(\alpha F_c(t, s; t_0, w, X, 0) + (1 - \alpha) F_c(t, s; t_0, w, X, \pi))
\]

\[0 \leq \alpha \leq 1\]

where \( Q \) is the sum of charges of these two clusters.

The two - cluster fitting function for a hit is chosen for better description of the signal’s back slope. The cathode output signal \( F_c(t, 0; t_0, 0, 0, \theta) \) from the strip number 0 for two values of \( \theta = 0; \pi \) and \( X = Y = 0 \) (the center of a chamber) are presented on the Figure 5. One can see that the signal has a long negative tail. It differs considerably from zero up to \( t = 10 - 15 \mu s \) but changes in time rather slowly. The contribution of any other cluster in the background for considered cluster is almost constant for given strip if only there is no temporal and spatial coincidence of two or more tracks.

To take into account the contribution of background, one more linear parameter will be used for each strip in the SCA shape.
Figure 5. The cathode output signal at the center of a chamber (ME3/1).

The Tables 3a, 3b present some typical contents of the SCA for one layer and the result of its fitting.

Further consideration is very similar to the procedure described in Section 3.2 for the search of the CLCT. The same kind of cathode envelope is applied to the hits parameters to find all possible SCA - LCTs and their allowed hits. The time distribution of the allowed hits is investigated to recognize the LCTs situated closely in space but visibly separated in time. In case of two or more allowed hits in the same layer only the one is chosen that suits best to the track created by allowed hits for all the other layers. Finally that LCT is selected which consists of not less than 3 hits and suits best to the pair \((b_i, h)\). For each so defined SCA - LCT one obtains 3 - 6 allowed hits. Each hit is characterized by its coordinates:

- the number of the layer containing the hit, 
- the origin time, 
- the wire group, 
- the charge of the hit, 
- the number of the half - strip (an estimate of the coordinate of the hit), 
- the angle \(\Phi\) in the plane \(X, Y\),

\[ \Phi = \Phi_n + \delta \Phi, \]

\(\Phi_n\) is the rotation angle of the strip \(n\) with reference to the center of the chamber, 
\(\delta \Phi\) - the angle position of the hit inside of the strip \(n\).

The angle \(\Phi\) is connected with \(X, Y\) - coordinates of the hit by the following relation:

\[ \tan(\Phi) = \frac{X}{\sqrt{X^2 + (R_o + Y)^2}}. \]

Here
$R_0$ is the distance from the center of a chamber to the strips crossing point.

For convenience of comparison with results for rectangular chambers, $\Phi$ and $\delta \Phi$ can be transformed to length units:

$$X = R_0 \Phi,$$

$$\delta X = R_0 \delta \Phi.$$  \hspace{1cm} (1)

### 4.2 The time resolution.

Our aim now is to determine the muon track origin time $t$. The input data for this are the origin times $t_1, t_2, \ldots$ of the SCA - LCT allowed hits. The temporal spread $\sigma$ of $\delta t = t_i - t$ is determined generally by fluctuations of the drift time of initial electrons to the anode wire. Due to dependence of the fitting function $F_\epsilon$ on the hit wire group $g$, the average value of $\delta t$ does not depend on the track position along the strip. To find the muon origin time, we follow here the well known procedure. Let us arrange all hits origin times so that

$$t_1 \leq t_2 \leq t_3 \ldots$$

A result of measurement $\hat{t}$ of the muon origin time $t$ can then be defined by the equality:

$$\hat{t} = t_k,$$

where $k$ can have any value $(1, \ldots, 6)$.

![Figure 6. The time resolution $\sigma$ as a function of $k$.](image)

What value of $k$ provides better time resolution? The one that gives the lowest value of $\hat{t} - t$ spread ($\sigma$). It follows from the Figure 6 that $k = 3$ or 4.

In our calculations $k$ was to be taken equal to 3.

As far as the average shift of $\hat{t} - t$ is not equal to 0, a correction was used to minimize the number of mistakes in determination of the track bunch.
We can improve the results for the cathode (Table 2) if we replace the CLCT coordinates \( b_c \) and \( h \) by those we have from fitting of the SCA - contents for given CLCT. This procedure reduces essentially the number of mistakes in cathode bunch determination. The part of tracks registered with true bunch identification \( (b = b_a = h) \) increases from 88.7\% to 96.4\% for ME3/1.

Unfortunately, this procedure can be performed really only for the RTs. It does not improve the track selection efficiency.

The number of RTs with correct bunch position \( (b_a = b) \) and created by not less than 3 hits from different layers will be now 96.8\% for ME3/1 and 95.1\% for ME3/2. Let the number of them be \( N' \). Only these RTs will be used in the next section for determination of spatial resolution. All other real muon tracks will be missed. In real experiment there will be necessary to analyze (and reject) some number of extra RTs that do not correspond to any real track.

The results presented by Figure 6 don’t differ essentially from those for the case of ENC = 0.

4.3 The spatial resolution. Preliminary considerations.

A good choice of the function to be used for fitting can simplify considerably the track coordinate determination. It can provide a small (or even zero) average bias of measured coordinate compared to the real one.

A rough estimation of the muon track coordinate was performed on-line in the comparator logic. In simulation it is to be done once more by fitting of the SCA contents. We do it finally by means of the procedure described in the Appendix.

Let \( X_n \) (or \( \Phi_n \)) be a real track coordinate of the muon with number \( n \). The coordinate of the hit in the layer \( k \) \( x_{k,n} \) \( (\Phi_{k,n}) \) is then its measured value. For each registered muon we have up to 6 \( x_{k,n} \). The best estimation \( \hat{x}_n \) of \( X_n \) can be calculated by means of the formula (A1) from the Appendix.

The average bias of the measurement in the layer \( k \)

\[
 f_k(X) = \{ x_{k,n} \}_X - X
\]

is not equal to 0.

Due to rotation of even layers, we have for \( f_k(X) \) the following relation:

\[
 f_k(X) = \begin{cases} 
 f(X), & k = 1, 3, 5; \\
 f(X - \frac{a}{2}), & k = 2, 4, 6.
\end{cases}
\]

Here

- \( a \) - is the average strip pitch,

- \( X \) - is the true coordinate of the track.

The behavior of the function \( f(X) \) is shown on the Figure 7. One can see that this shift is rather small. We have

\[
 |f(X)| < 0.007 a
\]

for both chambers and for all values of \( X \).
Due to the symmetry of experimental conditions, the reduced correlation matrix $D_x$ has only 3 independent elements $(D_x)_{1,1}$, $(D_x)_{1,2}$ and $(D_x)_{1,3}$ ($6$ different elements for each $X$).

The matrix $D_x$ (and $D$) is not diagonal. The level of non-diagonality characterized by parameters

$$
\gamma_1 = \frac{(D_x)_{1,2}}{\sqrt{(D_x)_{1,1}(D_x)_{2,2}}},
$$

$$
\gamma_2 = \frac{(D_x)_{1,3}}{\sqrt{(D_x)_{1,1}(D_x)_{3,3}}},
$$

is presented on Figures 8, 9 for both chambers.

The values of non-diagonal elements can be reduced for some extent if one takes into account the dependence of $f_k(X)$ on track position along the strip. We have totally ignored this dependence in this note.

One-layer spatial resolution $\sigma = \sqrt{D_{1,1}}$ as a function of $X$ is presented on the Figure 10. It differs radically from that one for the case when ENC of electronics is equal to zero. In this case the dependence of $D$ on $X$ almost vanishes (not shown here).

In a good approximation diagonal element of the reduced correlation matrix $(D_x)_{1,1}$ does not depend on the cluster charge $Q$. The Figure 14 (at the end of this paper) confirms that our parametrization of the weight factor (A9) of the matrix $D$ is good enough.
4.4 The spatial resolution. Final results.

Starting from this point, we cannot use in calculation our knowledge of the true track coordinate $X$. All details of this consideration are described in Appendix.

The correlation matrix $D$ has been re-calculated once more. Its diagonal element is close to that we had in the previous section. But now the matrix $D$ is assumed to be diagonal.

The average bias of the measurement in the layer $k$ $f_k^{(1)}(\hat{x})$ only slightly differs from $f_k(X)$. It turns out that the absolute value of average bias of a measurement $f^{(1)}(\hat{x}) = \hat{x} - \frac{1}{X_n} X_n$ for both chambers and for all values of track position $x$ does not exceed 20 $\mu$m (Figure 12). So we don’t need any corrections of measured coordinate $\hat{x}$ of a muon.

The calculation of $\hat{x}$ with (A1) leads to further reduction of the number of tracks ($N'_w$). The following two weak restrictions were imposed on the hits coordinates $x_{k,n}$ and $Q_{k,n}$ for each track:

$$20 fC < Q_{k,n} < 3500 fC,$$

$$|x_{k,n} - \hat{x} - f_k^{(1)}(\hat{x}_{n})| < 2500 \mu m.$$  \hspace{1cm} (2)

For ME3/1, 0.1% of tracks in all hits didn’t meet these requirements. For ME3/2 this part was 0.2%. The spatial coordinates for them are those determined from the analysis of the SCA contents with the envelope algorithm. The percentage of tracks ($N''_w$) that will be approved to be registered more strictly will be now 96.7% for ME3/1 and 94.9% for ME3/2.

Let $n(\Delta)$ be the part (from $N''_w$) of tracks that meets the condition:

$$|X - \hat{x}| \leq \Delta$$
In simulation one can calculate the RMS $\sigma$ for this part. The Figure 11 shows the dependence of $\sigma$ on $n = n(\Delta)$ for both chambers. It suggests the value $91 \, \mu\text{m}$ for the spatial resolution of ME3/1 and $103 \, \mu\text{m}$ of ME3/2 at $n = 99\%$.

The respective values for the case of $ENC = 0$ are $38 \, \mu\text{m}$ for ME3/1 and $62 \, \mu\text{m}$ for ME3/2.

From this one can conclude: the noise in electronics is the main source of degradation of the spatial resolution of our chambers.

The values of $\Delta$ and $\sigma$ for several values of $n$ and $n_l$ are presented in Tables 5,6. Here $n_l$ is the number of layers with hits involved in coordinate determination.

The spatial resolution calculated with the formula A2 from the Appendix gives the value $96 \, \mu\text{m}$ for the spatial resolution of ME3/1 and $98 \, \mu\text{m}$ of ME3/2 at $n = 99\%$. There are not any visible correlations between measurements in different layers of a chamber due to muon beam background influence.

5 Conclusions.

We can draw several conclusions from our simulation of the End Cap Muon System performance in realistic background conditions (with muon beam intensity $70 \, \text{Hz/cm}^2$).

Let $N_w$ be the total number of muons that have crossed the registration window of a chamber.

We have derived that

- The dependence of efficiency on background rates can not be neglected. Figure 2 shows decreasing of efficiency for ME3/1 from $99\%$ to $97.1\%$ when muon beam intensity grows.
Figure 10. One-layer spatial resolution $\sigma = \sqrt{D_{1,1}}$ as a function of $X$. from 10 Hz/cm$^2$ to 70 Hz/cm$^2$. We failed to find the main source of efficiency “leakage”. But some impression about this phenomena can be obtained from Table 2.

- The noise in electronics does not degrade essentially the trigger efficiency $E$. It was equal to 96.9% for ME3/1 and 95.3% for ME3/2.

- The noise in electronics does not degrade visibly the SCA time resolution (3.3 ns).

- Only 97.1% of muons for ME3/1 and 96.0% for ME3/2 had the time coordinate determined correctly by the trigger. All other muons were lost or registered with wrong bunch numbers. Unlike to the efficiency, we did not require true spatial coordinates for the tracks here.

- Let the total number of bunches selected by the trigger be $n_t$. The part of them with “false” muons registered by the trigger was equal to 4% ($0.04n_t$) for ME3/1 and to 3% ($0.03n_t$) for ME3/2.

- Only 96.9% (95.3%) of tracks were reconstructed by the trigger without mistakes.

- Only 96.8% (95.1%) of muons for ME3/1 (ME3/2) had the track coordinates evaluated from fitting of the SCA information.

- 96.7% of the tracks for ME3/1 and 94.9% for ME3/2 had the spatial coordinate that could be evaluated by means of statistical methods considered in the Appendix. These were the tracks that met the requirements (2) for one layer at least. We denote the number of them as $N_{tr}^{\sigma}$.

The precision of this procedure was 91 $\mu$m for ME3/1 and 103 $\mu$m for ME3/2 for 99% of tracks ($0.99 N_{tr}^{\sigma}$).
Figure 11. The spatial resolution $\sigma$ as a function of the number of events ($\%$).

At the same time (see details in Tables 4b, 5b):

- 98% of these tracks ($0.98 \ N_w$) had the spatial coordinate that could be evaluated with RMS $85 \mu m$ in case of ME3/1 and $90 \mu m$ for ME3/2.
- For 95% of tracks the spatial coordinate was known with RMS $77 \mu m$ (79 $\mu m$) for ME3/1 (ME3/2).

- The main source determining the spatial resolution of chambers is the noise of electronics.

Of course the real experimental conditions will be more severe. The gamma background, the muon halo, the influence of magnetic field can hinder the results presented above. The model developed here is apparently too detailed and requires much computer time for calculation. We need a more rough and fast but yet realistic model of the chambers performance that takes into account all these phenomena.
6 Appendix.

Some details of the procedure considered here can be found in [10].

Let $X_n$ be the real track coordinate of the muon with number $n$,
The vector $x_n$ and it’s transposed - $x_n^t$ will be it’s non-biased measurement (the estimate).

\[ (x_n)_k = x_{k,n}. \]

Here

$x_{k,n}$ is the measurement of $X_n$ in the layer $k$.

$l$ is the number of layers in the chamber ($l = 6$).

Some of the layers can be omitted.

Let us define the $l$ - dimensional vector $E$ ($E_k = 1$).

The vector $x_n$ can be used now for calculation of the best estimate $\hat{x}_n$ of $X_n$. We need for this the correlation matrix $D(X)$. If it has been already calculated, one has the following formula for $\hat{x}_n$:

\[ \hat{x}_n = E^t D^{-1}(X_n) x_n. \] (A1)

Let $l_n$ be the number of measurements for a given track $n$. The non-biased estimate of the dispersion $\hat{d}(\hat{x}_n)$ of $\hat{x}_n$ will be:

\[ \hat{d}(\hat{x}_n) = \frac{1}{l_n - 1} y_n^t D^{-1}(X_n) y_n. \] (A2)

Here

\[ y_n = x_n - \hat{x}_n E. \]

If for any layer $k$ there is no measured $x_{k,n}$, then the row and the column with this number in all the above formulas are deleted.

Because $\hat{d}(\hat{x}_n)$ is non-biased, we have the following equality for the true $d(X)$ and for the estimated $d_e(X)$ dispersions:

\[ d(X) = d_e(X) = \{\hat{d}(\hat{x}_n)\}_X. \]

Averaging of this equality over all tracks (over all $X$) gives the equality for full dispersions:

\[ d = d_e. \]

The formulas A1 and A2 are of the practical interest if only for each track substitution of $X$ by $\hat{x}$ is possible.

The correlation matrix $D$ can be written as

\[ D(X) = \{Y_n^t Y_n\}_X. \] (A3)
Here
\[ Y_n = x_n - X_n. \]

Let us define the two matrices:
\[ D^{(1)}(X) = \{y_n y_n^t\}_X \]
and
\[ D^{(2)}(\hat{x}) = \{y_n y_n^t\}_{\hat{x}}. \]

The matrix \( D^{(1)} \) can be easily calculated for any known \( D \), the matrix \( D^{(2)} \) is known from experimental data. If for each track one postulates that
\[ D^{(1)}(X) = D^{(2)}(\hat{x}), \]
then we can determine the \( D \) - matrix from \( D^{(2)} \). Both matrices have the same symmetry properties and have only 6 different elements. To find \( D \) from known \( D^{(2)} \) we need to solve the system of 6 non-linear equations for each \( X \).

Usually, \( D \) - matrix is diagonal. If it is the case, the system will have only 2 equations for 2 diagonal elements of \( D \). They can be solved easily. Let \( D_{1,1}^{(2)} \) and \( D_{2,2}^{(2)} \) be the diagonal elements of the matrix \( D \). \( D_{1,1}^{(2)} \) and \( D_{2,2}^{(2)} \) are the same for matrix \( D^{(2)} \). They are related by the following formulas
\[ D_{1,1} = \frac{D_{1,1}^{(2)}}{k_1}, \]
\[ D_{2,2} = \frac{D_{2,2}^{(2)}}{k_2}. \]

Let us denote
\[ k = \frac{D_{1,1}^{(2)}}{D_{2,2}^{(2)}}. \]

Then \( k_1 \) and \( k_2 \) will be:
\[ k_1 = \frac{1}{3} + \frac{1 + 8k}{3(\sqrt{k^2 + 7k + 1} + 3k)} \]
\[ k_2 = \frac{5}{3} - k_1. \]

Practically, the calculations of \( D \) can be performed by mean of some iteration procedure. In first approximation we take for \( D \) the unit matrix and calculate in first approximation \( \hat{x}_n \) for each track \( n \). With so determined \( \hat{x}_n \)'s we calculate \( D^{(2)} \) and from it the matrix \( D \) in second approximation and then the second approximation for \( \hat{x}_n \), and so on.

As far as \( D \) depends on \( X \), the calculation of \( \hat{x}_n \) in each approximation is also iterative. We take \( D \) for some value of \( \hat{x} = x_0 \) and calculate the new value of \( \hat{x} = x_1 \) with the formula (A1). Then the new value \( x_2 \) for \( \hat{x}_n \) is calculated with \( D(x_1) \). This procedure is performed while the difference between \( x_m \) and \( x_{m+1} \) exceeds some small number.

Generally, \( x_{k,n} \) is a biased estimation of \( X_n \) in a layer \( k \). This leads to some additional phenomena complicating the problem.

Let \( f_k(x) \) be the average bias of \( x_{k,n} \) for fixed \( X_n = X \):
\[ f_k(X) = \frac{\{x_{k,n}\}_X - X}{24} \text{ (A4)}. \]
Similarly $f_k^{(1)}$ is the average bias of $x_{k,n}$ for fixed $\hat{x}_n = \hat{x}$.

$$f_k^{(1)}(\hat{x}) = \overline{x_{k,n}} - \hat{x}. \tag{A5}$$

The same for $\hat{x}_n$ is

$$f^{(1)}(\hat{x}) = \hat{x} - \overline{X_n}.\tag{A6}$$

Suppose that the initial muon tracks are distributed uniformly across the strips with some constant density $p_0$. Then the distribution of measured track positions will be $p(\hat{x})$. Both these distributions and $f^{(1)}(\hat{x})$ are related by the equality:

$$f^{(1)}(x) = \int_0^x \frac{p_0 - p(x)}{p_0} dx. \tag{A7}$$

The Figure 12 demonstrates the quality of calculations with this formula. The agreement is rather good.

To obtain the non-biased measurement of $x_n$, one needs to perform the transformation of $x_{k,n}$ into $x^{[0]}_{k,n}$:

$$x^{[0]}_{k,n} = x_{k,n} - f_k(X_n). \tag{A7}$$

Then

$$\overline{x^{[0]}_n} = X.$$

Figure 12. Average bias $f^{(1)}(\hat{x})$ for ME3/1.

The problem is that such operation can be done only in simulation. Really we never know the value of $X_n$. Instead of this, one usually performs the transformation $x_{k,n}$ into $x^{[1]}_{k,n}$:

$$x^{[1]}_{k,n} = x_{k,n} - f_k^{(1)}(\hat{x}_n) - f^{(1)}(\hat{x}_n). \tag{A7}$$
These two transformations are not at all equivalent. They are even the functions of different variables,

\[ x_{k,n}^{(1)} - x_{k,n}^{(0)} = \gamma_{k,n} \neq 0. \]

In spite of this, we assume that \( x_{k,n}^{(1)} \) is a non-biased estimation of \( x_n \):

\[ \overline{\gamma_{k,n}} = 0. \]

One can now use so defined “non-biased” \( x_{k,n}^{(1)} \) and calculate \( \hat{x}_n \) and the dispersion \( \hat{d}_n \) with (A1, A2).

The estimation of the track position will be somewhat modified now. As usually, we take for \( D \) in first approximation the unit matrix and calculate in first approximation \( \hat{x}_n \) for each track \( n \). Then we perform the above discussed procedure and determine \( f^{(1)}(\hat{x}) \) in first approximation. Then we calculate the average bias \( f_k^{(1)}(\hat{x}) \) of \( x_{k,n} \) for fixed \( \hat{x}_n \) and for each layer \( k \) of the chamber. The formula (A7) permit now to eliminate the bias in each measurement \( x_{k,n} \) and \( \hat{x}_n \).

They are used for calculation of \( D_k^{(2)} \) in second approximation and then of the second approximation for \( D \), and so on.

In highly intensive beam we can meet another difficulty.

Let \( \beta = \beta_1, \beta_2, \ldots, \beta_n, \ldots \) be a set of numbers with their average value \( \overline{\beta} = 0 \).

It’s dispersion is \( \overline{\beta^2} = d_\beta \).

We define the vector \( B_n = \beta_n E \).

Performing the transformation of non-biased measurements \( X_n \) for all \( n \)

\[ D_{1,1} \text{ as function of cluster charge } q \text{ for ME3/1.} \]
\[ X^{(2)}_n = X_n + B_n, \]
we will have a new set \( X_n^{(2)} \) of non-biased measurements.

Such transformation does not influence on the \( D \) - matrix, but transforms all \( \hat{x}_n \):

\[ \hat{x}_n \to \hat{x}_n + \beta_n. \]

As it was before, the formula (A2) gives for the new estimated \( d_e^{(2)} \) the value:

\[ d_e^{(2)}(\hat{x}) = d_e(\hat{x}). \]

But now the estimate of \( d \) will be biased. We have for \( d^{(2)} \):

\[ d^{(2)}(\hat{x}) = \{ x_n - \hat{x}_n^{(1)} \} \beta + d_\beta(\hat{x}) = d_e(\hat{x}) + d_\beta(\hat{x}) \]

The last remark. In all the above formulas \( D \) was considered to be a function of \( X \) only. This is not very true. In fact \( D \) is a function also of the vector of cluster charges \( Q_n \) (Figure 13). For each track \( n \)

\[ Q_n = (q_n^{(1)}, q_n^{(2)}, \ldots, q_n^{(\theta)}), \]

We assume here that \( D \) can be presented in the form:

\[ D(X, Q) = D_e(Q) D_x(X) D_\beta(Q). \tag{A8} \]

Here

\( D_x(X) \) is the reduced correlation matrix,

\[ \text{Figure 14. } (D_x)_{1,1} \text{ as function of cluster charge } q \text{ for ME3/1.} \]
$D_c(Q)$ is the diagonal matrix, that performs the following operations with $x_{k,n}$:

$$D_c(Q)x_{k,n} = f_c(q_n^{(k)})x_{k,n}.$$ 

The function $f_c(q_n^{(k)})$ was taken in the form:

$$f_c(q) = \sqrt{c_0 + c_1 q + c_2 q^2 + \ldots}.$$  \hspace{1cm} (A9) 

The c’s here were adjusted to minimize the dependence of the reduced matrix element $D_z(\hat{x})_{1,1}$ on $Q$.

The Figure 14 illustrates the quality of our adjustment for case of ME3/1.
References

Table 1: The chamber parameters.

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<th>Parameter</th>
<th>ME3/1</th>
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<td>average cathode signal amplitude [fC]</td>
<td>110</td>
<td>120</td>
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</table>
Table 2: The map of best (ALCT,CLCT) representatives for all GTs.

| $b_a - b$ | $b_c - b$ | $|g_a - g|$ | $|h_c - h|$ | statistics | (%)  | acc. (%) |
|-----------|-----------|-------------|-------------|------------|-------|----------|
| 0         | 0         | 0           | 0           | 23029      | 77.4  | 77.4     |
| 0         | 0         | 0           | 1           | 2259       | 7.6   | 85.0     |
| 0         | -1        | 0           | 0           | 1044       | 3.5   | 88.6     |
| 0         | 1         | 0           | 0           | 1013       | 3.4   | 92.0     |
| 0         | 0         | 1           | 0           | 1008       | 3.4   | 95.4     |
| 0         | -1        | 0           | 1           | 187        | 0.6   | 96.0     |
| 0         | 2         | 0           | 0           | 139        | 0.5   | 96.4     |
| 1         | 0         | 0           | 0           | 106        | 0.4   | 96.8     |
| 0         | 0         | 0           | 0           | 99         | 0.3   | 97.1     |
| 0         | 0         | 1           | 1           | 94         | 0.3   | 97.5     |
| 0         | 0         | 0           | 0           | 93         | 0.3   | 97.8     |
| 0         | 1         | 0           | 1           | 82         | 0.3   | 98.0     |
| 0         | -1        | 1           | 0           | 47         | 0.2   | 98.2     |
| -1        | 0         | 0           | 0           | 39         | 0.1   | 98.3     |
| 0         | 3         | 0           | 0           | 34         | 0.1   | 98.4     |
| 0         | 1         | 1           | 0           | 32         | 0.1   | 98.6     |
| 2         | 0         | 0           | 0           | 28         | 0.1   | 98.6     |
| 0         | -2        | 0           | 1           | 21         | 0.1   | 98.7     |
| 1         | 0         | 1           | 0           | 19         | 0.1   | 98.8     |
| 1         | 0         | 0           | 1           | 16         | 0.1   | 98.8     |
| 0         | -3        | 0           | 1           | 15         | 0.1   | 98.9     |
| 0         | 2         | 0           | 1           | 15         | 0.1   | 98.9     |
| 1         | 0         | 0           | 0           | 14         | 0.0   | 99.0     |
| 0         | 1         | 0           | 10          | 13         | 0.0   | 99.0     |
| 0         | 1         | 0           | 10          | 12         | 0.0   | 99.1     |
| 0         | -2        | 0           | 0           | 12         | 0.0   | 99.1     |
| 0         | -3        | 0           | 2           | 11         | 0.0   | 99.1     |
| 0         | -1        | 0           | 2           | 11         | 0.0   | 99.2     |
| 0         | 2         | 1           | 0           | 11         | 0.0   | 99.2     |
| 2         | 0         | 1           | 0           | 11         | 0.0   | 99.3     |
| all       | other     | 221         | 0.7         | 100.0      |       |          |

all statistics \(29735\) 100.0 100.0
Table 3a: The SCA - contents for some CLCT before fitting.

**event 40 layer 3**

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<td>-0.1</td>
<td>-0.1</td>
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<td>-0.1</td>
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Table 3b: The SCA - contents for some CLCT after fitting.

**event 40 layer 3**

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Table 4a: The $\Delta(n) = |\hat{x} - X|$ distribution for ME3/1.

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<tr>
<th>$n_l$</th>
<th>statistics</th>
<th>$\Delta(100%)$ [(\mu m)]</th>
<th>$\Delta(99%)$ [(\mu m)]</th>
<th>$\Delta(98%)$ [(\mu m)]</th>
<th>$\Delta(95%)$ [(\mu m)]</th>
<th>$\Delta(90%)$ [(\mu m)]</th>
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<tbody>
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<td>8239.045</td>
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<td>8564.741</td>
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<tr>
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<td>265.971</td>
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Table 4b: The $\sigma(n) = \sigma(\hat{x} - X)$ distribution for ME3/1.

<table>
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<th>$n_l$</th>
<th>statistics</th>
<th>$\sigma(100%)$ [(\mu m)]</th>
<th>$\sigma(99%)$ [(\mu m)]</th>
<th>$\sigma(98%)$ [(\mu m)]</th>
<th>$\sigma(95%)$ [(\mu m)]</th>
<th>$\sigma(90%)$ [(\mu m)]</th>
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<tr>
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<td>4924.958</td>
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<td>3947.151</td>
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<tr>
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<td>3426.809</td>
<td>3409.666</td>
<td>2769.366</td>
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<tr>
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<td>239</td>
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<td>541.093</td>
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Table 5a: The $\Delta(n) = |\hat{x} - X|$ distribution for ME3/2.

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<td>3309.516</td>
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<td>649.862</td>
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<td>509.233</td>
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<td>307.457</td>
<td>246.813</td>
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Table 5b: The $\sigma(n) = \sigma(\hat{x} - X)$ distribution for ME3/2.

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<th>statistics</th>
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